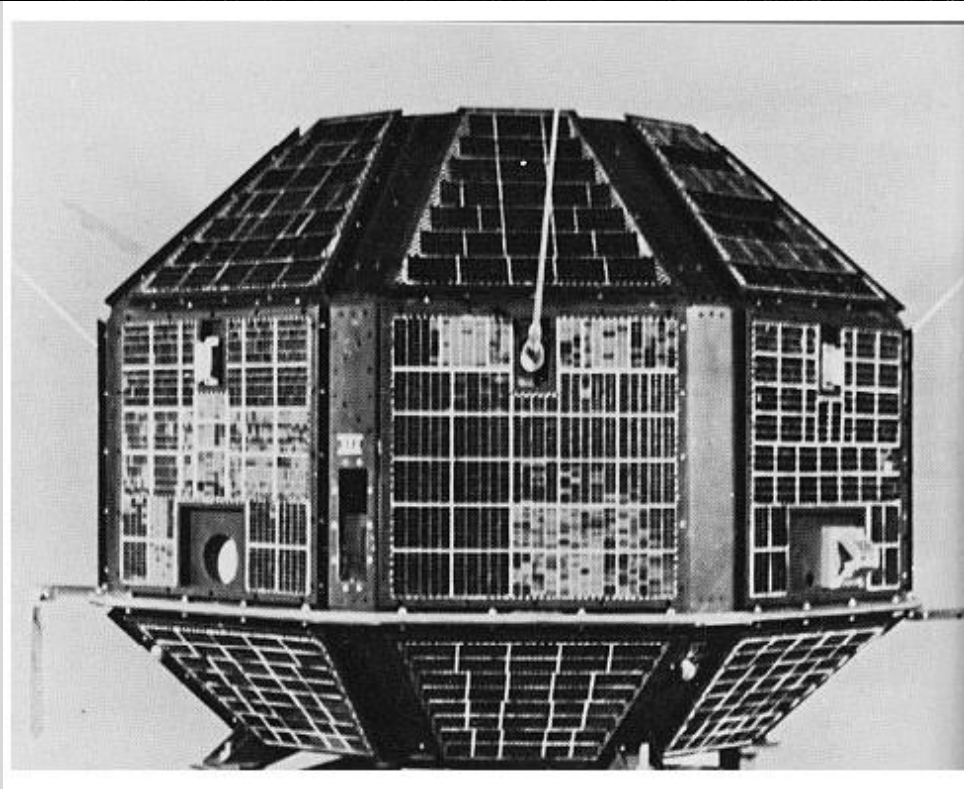
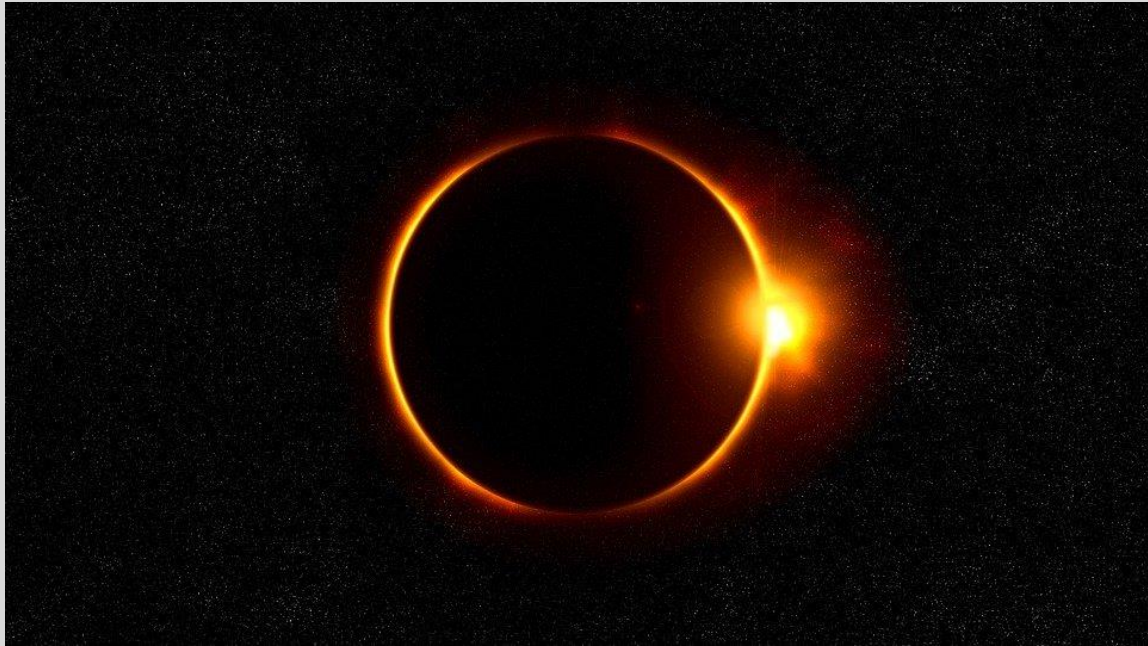


A Brief on ARYABHATIYA



Book by: Kandhadai Madhusoodhanan Ramakrishnan

A Brief on ARYABHATIYA

Chapter I. Aryabhata- The Author:

Aryabhata is a composition of Aryabhata. We all know his name very well. However, we know very less about him as he, himself has not shared much about him. We have few information about him through his compositions. In Aryabhata, he has mentioned his name twice, first in opening stanza of first chapter of Gitika pada and in opening stanza of second chapter of Ganita pada. We will see all those chapters in details as we go on.

We all know that, first Indian satellite was designated “Aryabhata” after his name and was put into orbit on April 19, 1975. Time is an important dimension which places an important role in all our life. Any situation or a state of person or state of any object cannot be defined without defining appropriate time for the same. For defining any timeline, we need a reference which can define starting point. Ideally this reference of Time Zero, should have been the time when first universe got created for the first time or a definite point in time as reference, if time is considered Anadi (no beginning and end). Also, for measurement of time, we need something which has a periodicity and which ideally doesn't change its periodicity with respect to time scale. For example, let us take our age. Our age is actually a measurement of number of years we lived after our birth. Time taken for earth to revolve once around sun is defined as a year. Interestingly, our age is actually number of times we revolved around sun. Every living being, has an inbuilt clock inside then which is defined based on Sun and Moon pattern. Let us go in-depth on these scales defined by Aryabhata as we go on inside Aryabhata from Chapter II.

Year of birth of Aryabhata is shared by Aryabhata himself with precision. There is a verse in Aryabhata which describes as “When sixty times sixty years and three quarter-yugas had elapsed, twenty-three years had then passed since my birth”. From the above verse, it is clear that Aryabhata was twenty-three years of age and it is Kali year 3600. As per modern time reference, Kali year 3600 corresponds to A.D. 499. So, Aryabhata was born in year A.D. 476 ($499-23=476$). To be more precise, 3600 years of Kali yuga came to an end on Sunday, March 21, A.D. 499, at mean noon, at the time of Mean sun's entrance into the sign Aries (Madhyama-mesa-sankranti). So, the time of birth of Aryabhata is on Mesa-sankranti on March 21, A.D. 476. Reference date given here is on Modern calendar reference.

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1. Do you know?

Ancient works in the field of Astronomy by Indians: (Listed only few)

Apastamba sulba sutra
Aryabhatiya
Atharvana jyotisa
Bhakhshali
Baudhayana Sulba sutra
Bijaganita
Brahma sphuta siddhanta
Bijaganitavatamsa
Brhajataka
Brhadjotishasara
Brhatsamhita
Candraprajapati
Dhikotidakara
Dhivrddhida tantra
Drgganita
Ganitakaumudi
Ganitamuyoga
Ganitasarasangraha
Ganitatilaka
Goladipika
Golasara
Grahasaranibandana
Grahalagava karana
Grahana mandana
Grahana nyayadipika
Grahanastaka
Jambudvipa prajnapiti
Jambudvipa samasa
Jothisha siddhanta sangraha
Jothiscandrarka
Karana kaustubha
Karana kutuhala
Karanamrta
Karana paddhati
Karana prakasa
Karana ratna
Karanendusekhara
Karanottama
Kasyapa samhita
Katyayana sulba sutra
Khanda khadyaka
Kuttakara siromani
Laghu bhaskariya
Laghumanasa

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Lilavati
Maha bhaskariya
Maha siddhanta
Manava Sulba sutra
Pancasiddhantika
Patiganita
Rajamrganita
Rasigolasphutaniti
Rekhaganitha
Romaka siddhanta
Sadratnamala
Sarvananda karana
Sarvananda laghava
Siddhanta dharpana
Siddhanta sarvabauma
Siddhanta sekera
Siddhanta siromani
Siddhanta sundara
Siddhanta tattvaviveka
Sishya dhi vrddhida
Soma siddhanta
Sphutacandrapti
Sphutanirnaya tantra
Suryaprajnapti
Surya siddhanta
Tantra sangraha
Tattvarthadhigama sutra bhasya
Tiloya pannatti
Triloka sara
Trisatika
Vakyakarana
Vatesvarasiddhanta
Vedanta jyotisa
Venvaroha
Visva prahelika
Yuktibhasa

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2. Do you know?

Ancient Indian multiplication:

Common Core Math

$8439 \times 584 = ?$

8 4 3 9

4	4	2	1	4	5
9	6	3	2	7	8
2	3	1	1	3	4
8	3	7	6		

4,928,376

Standard Multiplication:

$$\begin{array}{r} 8439 \\ \times 584 \\ \hline 33756 \\ + 67512 \\ + 42195 \\ \hline 4928376 \end{array}$$

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Chapter II. Aryabhatiya – Introduction

Aryabhatiya deals with both Mathematics and astronomy. It contains 121 stanzas. It is divided into four chapters called Padas.

Pada 1 - Gitika-Pada: Consists of 13 stanzas which includes basic definitions & important astronomical parameters and tables. It gives definitions of larger unit of time (Kalpa, Manu and Yuga), the circular units (Sign, degree and minute) and the linear units (Yojana, nr, hasta and angula) and states the number of rotations of the Earth and the revolutions of the Sun, Moon and planets, etc., diameters of Earth, Sun, Moon and other planets, etc.,

Pada 2- Ganita Pada: Consists of 33 stanzas, which deals with mathematics. This pada deals with geometrical figures, their properties and mensuration.

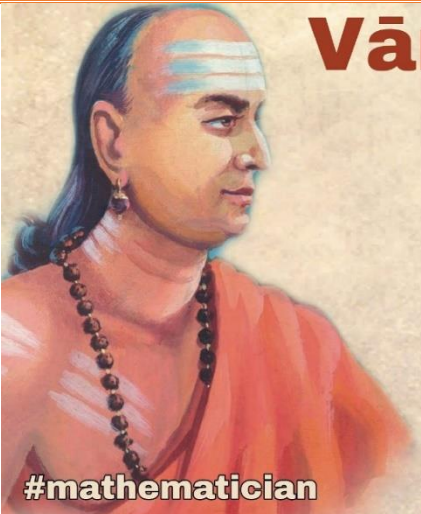
Pada 3- Kalakriya Pada: Consists of 25 stanzas, which deals with various units of time and determination of true position of Sun, Moon and planets.

Pada 4- Gola Pada: Consists of 50 stanzas, which deals with motion of Sun, Moon and planets.

Next, we will see Gitika pada sutras starting with Mangalacaranam & Vedic number system.

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3. Do you know?



Vārāhamihira
around 6 CE
4×4 Magic Square

2	3	5	8
5	8	2	3
4	1	7	6
7	6	4	1

#mathematician

4. Do you know?

Testimonies from two French mathematicians:

- “The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of Antiquity, Archimedes and Apollonius.” — Laplace (early 19th century)
- “The Indian mind has always had for calculations and the handling of numbers an extraordinary inclination, ease and power, such as no other civilization in history ever possessed to the same degree. So much so that Indian culture regarded the science of numbers as the noblest of its arts.... A thousand years ahead of Europeans, Indian savants knew that the zero and infinity were mutually inverse notions....” — Georges Ifrah (1994)

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Chapter III. Gitika Pada

Stanza 1:

प्रणिपत्य एकं अनेकं कं सत्यां देवतां परं ब्रह्म ।
आर्यभटस् त्रीणि गदति गणितं कालक्रियां गोलम् ॥ १.१ ॥

“Having paid obeisance to God Para-Brahmam – who is one and created many, the real God, the Supreme Brahmam- Aryabhata sets forth the three, viz., mathematics (Ganita), reckoning of time (Kalakriya) and celestial sphere (Gola)”

This stanza is Mangalacaranam of Aryabhata to Supreme Brahmam.

In above stanza, text "kam" can be interpreted as "anandakam" (supreme bliss), "satyam" as "satsvarupam"(really existing truth for ever) and "devatam" as "cit-svarupam" (pure intelligence). Para-Brahman, who was the only one initially, who created other gods and other forms, is the supreme bliss, real existing truth and pure intelligence. Aryabhata sets forth Ganita, kalakriya and Gola to the Para-Brahman as the mysteries of the science were revealed to Aryabhata after worshipping him.

Stanza 2:

वर्गाक्षराणि वर्गेऽवर्गेऽवर्गाक्षराणि कात् झुयै यः ।
खद्विनवके स्वरा नव वर्गेऽवर्गे नवान्यवर्गे वा ॥ २

“Varga letters ('k' to 'm') in the varga places and avarga letters ('y' to 'h') in the avarga places. (The varga letters take the numerical values 1,2,3, etc.,) from 'k' onwards; (the numerical values of the initial avarga letter) 'y' is equal to 'n' plus 'm' (i.e., 5+25). In the places of the two nines of zeros (which are written to denote the notational places), the nine vowels should be written (one vowel in each pair of the varga and avarga places). In the varga (and avarga) places beyond (the places denoted by) the nine vowels too (assumed vowels or other symbols should be written, if necessary)”

As mentioned earlier, Gitika section is very important as it set forth the definition and rules for further sections. It may seem that lots of information is given in one stanza, and it is true, it is also important to understand it thoroughly to understand other chapters. Things may seem complex as it is different but our normal thought process. I don't know Sanskrit and but managed to understand by referring some references. O will share in an easy way to understand. These are basic definition which can be understood if you know the letters. In our modern mathematics, there are 10 letter 0-9 with which we do all mathematics calculation. In vedic number system, there are more letter to represent counts. This helps in representing large numbers with lesser letters. For example, value of चौ (cau) is 6000000000000000 (6x10¹⁶). As Aryabhataiya is about astronomy, we will be seeing large numbers in upcoming stanzas. Nonetheless, vedic number system can and is also used for fractions too.

In Sanskrit alphabet, the letter 'k' to 'm' are classified into five vargas (classes) – ka-varga, ca-varga, ṭa-varga, ta-varga and pa-varga. Each varga classes has five letters. These letters will represent numerical values 1 to 25 as given below: (Please note Consonant(क) + vowel (अ) will give a syllable (क), which has a value)

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ka-varga: क (k) =1; ख (kh) =2; ग (g) =3; घ (gh) =4; ङ (ñ) =5;

ca-varga: च (c) =6; छ (ch) =7; ज (j) =8; झ (jh) =9; ञ (ñ) =10;

ṭa-varga: ट (ṭ) =11; ठ (ṭh) =12; ड (ḍ) =13; ढ (ḍh) =14; ण (ṇ) =15;

ta-varga: त (t) =16; थ (th) =17; द (d) =18; ध (dh) =19; न (n) =20;

pa-varga: प (p) =21; फ (ph) =22; ब (b) =23; भ (bh) =24; म (m) =25;

In modern number system (10 digits decimal system), place of a number gives different value to represent large numbers. Example, for number 23 (reading from right, 3 is first and then 2 in second place) value of the number is actually $3 \times 10^0 + 2 \times 10^1$, which is $3 + 20$. Another example, for number 131, value of the number is $1 \times 10^0 + 3 \times 10^1 + 1 \times 10^2$, which is $1 + 30 + 100$. Here, to represent higher values, avarga letters and vowels are used.

The letters य (y) to ह (h) are called avarga letters. Numerical values of these letters are given below:

य (y) = 30; र (r) = 40; ल (l) = 50; व (v) = 60; श (ś) = 70; ष (ṣ) = 80; स (s) = 90; ह (h) = 100.

Please note that every avarga is increasing by 10 in values starting from 30, till 25 is covered in vargas.

We saw earlier that Consonant(क) + vowel (अ) will give a syllable (क), which has a value. To arrive at value, we should know the power of vowels. In last post, we saw value of क as 1, which can also mean that क (1) $\times$ अ (10^0), i.e., 1. Multiplication factor of vowels are given below:

अ (-a) = 10^0 ; इ (-i) = 10^2 ; उ (-u) = 10^4 ; ऋ (-r) = 10^6 ; ए (-l) = 10^8 ; ए (-e) = 10^{10} ; ओ (-o) = 10^{12} ; ऐ (-ai) = 10^{14} ; औ (-au) = 10^{16} ;

In above, short vowels are extensively used in Aryabhatiya. Long vowels (last two) are rarely used. Words which represents a value to be read from left to right. We will see with examples:

1. कृ = क (1) $\times$ ऋ (10^6)

= 1000000

2. षि = 80 $\times 10^2$

= 8000.

3. ख्युघृ = खु + यु + घृ

= $2 \times 10^4 + 30 \times 10^4 + 4 \times 10^6$

= 20000 + 300000 + 4000000

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= 4320000. This is the number of years earth has revolved around sun since the beginning of this Yuga. Yuga here is yuga cycle, including all small yugas like Krta, Treta, Dvapara and Kali, which are part of the cycle. We will see what is Yuga, Manu and Kalpa in time scale later.

Try converting value 108. You should get जकि. With this second stanza, which is on number system is over.

Refer Aryabhata Number system file for reference.

1. Interesting thoughts

Any event has two outcome based on the state of change:

1. Agreement and 2. Disagreement.

It can said that our existence is due to disagreement of matter with antimatter. Also, due to agreement of matter with another matter. Our body is healthy when it agrees in combining with another matter. When it disagrees, we get sick.

Mostly, we value matter/ objects in the world because we like agreement. We value time due to disagreement. We never get satisfied with the happiness what we get out of the agreement of matters. We age with time due to disagreements of our body with other matter. We never get satisfied of disagreement either. What do we actually want then? An insect stuck in a spider web would be happy when it can easily feed on another small insects which get stuck in that web. When spider (time) comes we only pray for spider not to come to us. Ultimately, we fail to ask for the freedom from web.

Stanza 3:

युगरविभगणास् ख्युघृ शशि चयगियिडुशुछलृ कु डिशिबुणूष्व प्राक् ।
शनि दुङ्विघ्व गुरु ख्रिच्युभ कुज भदलिङ्नुख् भृगुबुधसौरास् ॥ १.३ ॥

"In a yuga(युग) of 4320000(ख्युघृ) of solar (Sun- रवि) years, the eastward(प्राक्) revolutions of the Earth(कु), 1582237500(डिशिबुणूष्व); of the Moon(शशि), 57753336(चयगियिडुशुछलृ); of Saturn(शनि), 146564(दुङ्विघ्व); of Jupiter(गुरु), 364224(ख्रिच्युभ); of Mars(कुज), 2296824(भदलिङ्नुख्); of Mercury(बुध) and Venus(भृगु), the same as those of the Sun"

As we have seen number system in last stanza, there are few numbers shared in this stanza for us to understand. However, there are many ongoing researches by scholars on this stanza. I will share what I understood. These are also shared in details in appropriate chapters.

Revolutions mentioned in the above stanza are side real rotation about equator (it is mentioned in next stanza which is continuation of this one). One solar day is 24 hrs but one side real day is not exactly 24 hrs. Let us take only earth and sun related data in above stanza for understanding. In a yuga, earth rotates 1582237500 times in solar years of 4320000.

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Earth rotates about its axis (this axis is at 23.5deg inclination from polar axis) and rotates around sun (axis of this rotation is about north pole and south pole). Remember, I mentioned that these data are with reference to equator. We all know that there are places above arctic circle (which is an imaginary circle made by the 23.5 deg inclination reference) where sun doesn't set for approximately six months.

One rotation around sun is termed as one year. Aryabhata calls Sun as Day-Maker as he considered Sun defines the definition of a day. Number of days in a solar year can be calculated as $((1582237500 - 4320000)/4320000) = 365.2586805555556$, which is, 365 days, 6 hrs, 12 min, 30 seconds. Modern day value is 365 days, 6 hrs, 9 min, 9 sec. There are correction factors to arrive at more precise value. We will see those in relevant chapters.

Also, as earth rotates 1582237500 times and considering that it rotates once around Sun a year. Mean solar days for this period is $1582237500/4320000 = 1577917500$ days. Number of hours in a side real day is $(1577917500/1582237500) \times 24 \text{ hrs} = 23.93447254283886 \text{ hrs}$, which is, 23 hrs 56 min 4.1 seconds. I am leaving this value till accuracy level of a Prana (1 Prana is 4 seconds, which is the time taken for one breath of average human being). Modern value of this side real day is 23 hrs 56 min 4.091 seconds.

It is to our surprise that the accuracy of this data shared by Aryabhata is exact to the level of a Prana considered by Aryabhata. Aryabhata has not shared the source of this data as he would have got these through Veda teachings from his Acharya, with blessing of Para-Brahmam, God himself, as mentioned in Stanza 1. Earlier days, students believed the words of his acharya and questioned him to learn more on the subject. We struggle to do so as we were bound to complete a course in a constrained period of time. We didn't ask our teacher about "How an object gets its mass?" and "What is influencing gravity?".

We will see about Moon and other planets next.

Before Nicolaus Copernicus (1473–1543) idea that planets revolved around Sun, Modern science believed that Sky revolved around the Earth. However, Aryabhata shared that planets revolved around Sun, and shared both Heliocentric and Geocentric concept. Also, he shared that the Earth is sphere and follows an elliptical orbit around Sun.

Next let us take Moon, based on which month period is defined. Aryabhata shared that 1582237500 rotations of the Earth equal 57753336 lunar orbit. So, from this Lunar orbit per rotation of Earth can be calculated as $(1582237500/57753336) = 27.3964693572$. So, lunar orbit duration in number of side real days is $(27.3964693572 \times 23.93447254283886(\text{refer last post}))/24 = 27.32166848335957$, which is 27 days, 7 hrs, 43 min, 12 seconds. This value matches the correct value by seven digits (27.3216638). Again, accuracy level matches till Prana level (4 seconds).

For other planets, duration of their orbit in terms of side real period in terms of days are Mars- 686.99974; Jupiter – 4332.27217; Saturn – 10766.06465;

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In next post, we will see Heliocentric number of rotations in a Yuga, which is mentioned at the end of this stanza as same as that of Sun (सौरास्). This denotes Heliocentric number of revolutions in Yuga. We will also, see Apogee and Ascending Node of Moon in next post.

The orbital path of Moon is not uniform elliptic. Let us consider that it is elliptic around earth to understand Apogee. As it is elliptic, there will a point in orbit where Moon is far from Earth, which is called as Apogee (As this point, Moon will look smaller). The other point, where it is close to Earth is called Perigee (As this point, Moon will look bigger).

There are two lunar nodes, Ascending node & Descending node. These are called Rahu (Ascending node) & Ketu (Descending node) here. The lunar nodes are the two points where the Moon's orbital path crosses the ecliptic, the Sun's apparent yearly path on the celestial sphere. When they are aligned, we get to see an eclipse.

Stanza 4:

चन्द्रोच्चर्जुष्विध बुध सुगुशितृन भृगु जषबिखुछ शेष अर्कास्।
बुफिनच पात-विलोमास्बुधाहि अजार्कोदयात् लङ्कायाम्॥

(This stanza is a continuation of the earlier and forms one sutra with previous stanza)

"of the Moon's apogee(चन्द्रोच्च), 488219(र्जुष्विध); of (the sighrocca {heliocentric} of Mercury(बुध), 17937020(सुगुशितृन); of (the sighrocca{heliocentric}) Venus(भृगु), 7022388(जषबिखुछ); of (the sighroccas {heliocentric}) the other planets, the same as those of the Sun; of the Moon's ascending node in the opposite direction (i.e., westward), 232226; These revolutions commenced at the beginning of the sign Aries on Wednesday (Start of Yuga) at sunrise at Lanka"

Coincidentally, Today, we have solar eclipse is few parts of the world and we are going to see Ascending node (Rahu) of Moon. For our location in Thane, on Sarvari, Aani, 7, partial solar eclipse (based on modern time reference) will start from 10.01am and end at 1.28pm. Maximum eclipse will be at 11.38am.

Before going into number, let us see the importance of Lanka (लङ्कायाम्) mentioned in the stanza. Lanka here denotes Equator. Aryabhata shared Aryabhatiya at Ujjain, which passes through the Tropic of Cancer line. Location here is to be noted and remember as we will go in depth when we see how Aryabhata would have calculated Diameter of Earth to an accuracy level of 0.1%. For your information, As Earth's axis of rotation about itself is tilted by 23.5 deg (approx..) and also rotates around the Sun. Perfect noon, i.e. Sun exactly above us can be seen only in locations within Tropic of cancer to Tropic of Capricorn. For easy reference, when Sun is exactly above us, we won't be able to see shadow of object or person below Sun.

As explained in my last post, Moon's Apogee, is the remotest point on the elliptic path of Moon from earth. Number of side real days for Moon to complete one cycle from Apogee to Apogee is $(4320000/488219) \times 365.25868 = 3231.98708$ days. Modern value is 3232.37543 days.

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Moon's Ascending node (Rahu), is that point of the ecliptic, where the Moon crosses it in its northward motion. When Sun's position matches with this node, then we get to see Complete Solar eclipse. When Sun is behind Earth (where it is day for the other portion of Earth) and Moon crosses its Node exactly opposite to Sun, we get to see Complete Lunar Eclipse. Number of side real days for Moon to complete one cycle from Ascending node (Rahu) to Ascending node (Rahu) is $(4320000/232226) \times 365.25868 = 6794.74951$ days. Modern value is 6793.39108.

Values mentioned by Aryabhata are based on approximation made on many factors. Remember, there are reason why our teacher asked us to use 22/7 or 3.14 for value of Pi in our maths class and not exact long value with all decimal places. Our ancestors are very specific on accuracy level. We also have Pi value with 28 decimal places mentioned in Rig Veda. Katapayadi system of Number representation which may have come from Aryabhata number system gives value of Pi till 14 decimal places.

Next, we will see Heliocentric rotation of other planets mentioned in this stanza.

Aryabhata shared Aryabhata in 498 BC in University of Nalanda, in Patna. This was then translated into Latin in 13th century. Through this translation, European mathematician got to know the methods of calculating the areas of triangles, volumes of spheres as well as square and cube root. He was also the earliest to discover that the orbits of the planets around the Sun and ellipses. Aryabhata gives numerical and geometric rules for eclipse calculations. Einstein once said, *"We owe a lot to the Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made"*.

Back to our stanza, based on the number of geocentric rotation of Planets, Mercury(बुध), 17937020(सुगुशिथून) & Venus(भृगु), 7022388(जषबिखुछ), we can calculate the side real period in term of earth days. For Mercury, $(4320000/17937020) \times 365.25868 = 87.96988$ days. Modern value is 87.9693 days. For Venus, $(4320000/7022388) \times 365.25868 = 224.69814$ days. Modern value is 224.7008 days.

It can be seen from the above two stanzas that Mercury and Venus are represented in Heliocentric number of rotations and also for Moon apogee and ascending Node. These are represented in difference reference as compared to Jupiter and Saturn. Only Mercury, Venus and Moon can come in between Earth and Sun to create Solar eclipses. This stanza is over.

All modern values mentioned are with reference to Epoch J2000.0 reference. These values vary with time as Earth's self rotation speed is reducing due to Moon's gravitational force on tidal waves dragging it to slow down. This will increase the side real day time. Aryabhata shared these values based on his observation during his time. Next, let us see Time scale considered by Aryabhata based on Yuga cycle of 4320000 years.

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5. Do you know?

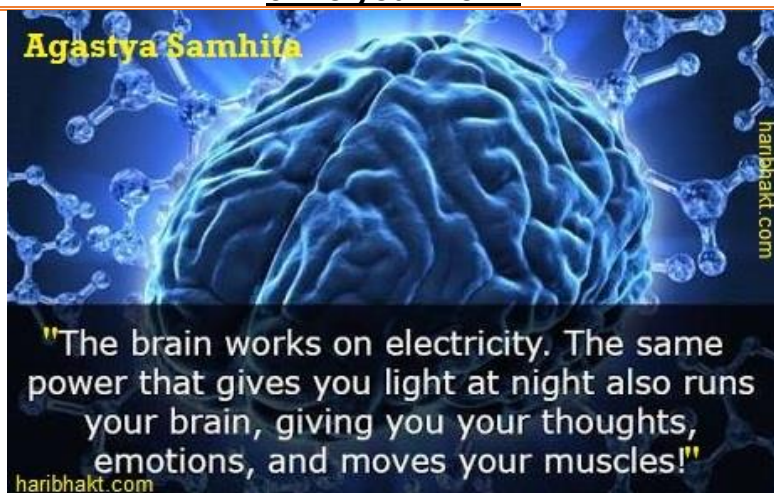
Siddhas or scientists?



- Agasthiyar is considered to be the author of a lot of the first Siddha literature and he was supposed to have lived in the 7th century BC.
- About 96 books are attributed to him and that includes writings in alchemy, medicine and spirituality.
- Apart from the legends that exist, the beginnings of the Siddhars' are lost in time.

6. Do you know?

Agastya Samhita

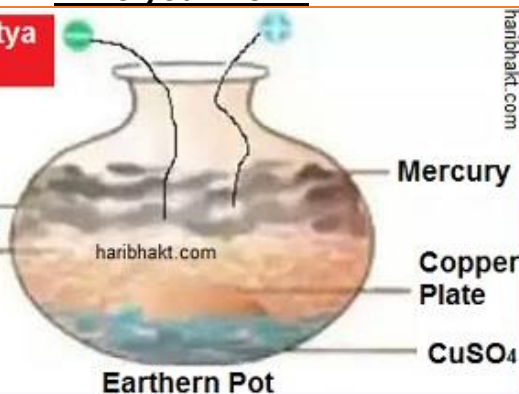


7. Do you know?

Rishi Agastya Battery

Zinc Powder
Wet Saw Dust

haribhakt.com



Mercury

Copper Plate

CuSO_4

Earthen Pot

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Stanza 5:

काहो मनवो ढ, मनुयु गाः शख, गतास्ते च, मनुयुगाः स्छना च ।
कल्पादेस्युगपादा ग च, गुरुदिवसाच्च, भारतात् पूर्वम् ॥

“A day of Brahma (or a Kalpa) is equal to (a period of) 14 Manus, and (the period of one) Manu is equal to 72 yugas. Since Thursday, the beginning of the current Kalpa, 6 Manus, 27 Yugas and 3 quarter Yugas had elapsed till the to end Bharata”

From the above stanza, we have, 1 Kalpa = 14 Manus; 1 Manu = 72 Yugas. So, 1 Kalpa = 1008 Yugas or 4354560000 years. These are Time scale above Yugas.

Second part of the stanza explain the Point of our current Yuga in a Brahma day cycle. The time elapsed since the beginning of the current Kalpa up to the beginning of the current Kaliyuga is 6 Manus + 27 $\frac{3}{4}$ Yugas, which is $(6 \times 72 + 27 \frac{3}{4})$ Yugas, ie., $(432 + 27 \frac{3}{4}) \times 4320000$ Years equals 1986120000 years (725447570625 days). Day of the current Kalpa started on Thursday. Current Manu is Shraddhadeva Manu or Vaivasvata Manu.

$725447570625 / 7 = 103635367232$ weeks and 1 day. This shows that Mahabharata purana ended (bharatat purvam) on Friday. Full Yuga cycle is considered here as one cycle, so, it is also to be noted that there is an overlapping phase between the small Yuga. Hence, Aryabhata did not specify end of Mahabharata as starting of Kali Yuga. It is also to be noted that inbetween each small yugas, there is an overlapping phase called Yuga Sandhi. Hence, complete Yuga (including all four small yugas) is considered to form one Yuga cycle.

Stanza 6:

शशिराशयस् ठ चक्रं तेऽशकलायोजनानि य-व-ज-गुणास् ।
प्राणेनैति कलां भं, खयुगांशे ग्रहजवो, भवांशेऽर्कस् ॥ १.६ ॥

“Reduce the Moon’s revolutions (in a Yuga) to rāśa (राश, signs), using the fact that there are 12(ठ) signs in a revolution. Those signs multiplied successively by 30(य), 60(व) and 10(ज) yield tēśa(तेऽश,degrees), kalā (कला,minutes) and yojanas(योजना), respectively. Earth rotates through one minute of arc in one prana (प्राणेनैति कलां भं). The circumference of the sky divided by the revolutions of a planet in a yuga gives the orbit on which the planet moves. The orbit of the constellations 60 times the orbit of the Sun”.

From the first line on yojanas, minute and degrees division, it is clear that 10 yojana = 1 arc minute; 60 arc minute = 1 degree; Moon’s revolution is divided into 12 signs. Which means that 1 revolution = 12 x 30 degree = 360 degrees.

From the second line, the time prana is the time taken by Earth to rotate by 1 arc minutes, which can be calculated as

For 1 side real revolution, 360 degree = 23.93447254283886 hrs

So, 1 arc minute = 23.93447254283886hrs x 60x 60 /360/60

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$$= 0.0664846459523302 \times 60 \times 60 / 60$$

$$= 3.989 \text{ side real seconds}$$

So, 1 prana (1 respiration) is roughly considered as 4 seconds.

Next, circumference of the sky divided by revolutions of a planet in a Yuga gives the orbit on which the planet moves. This is for circumference of orbital motion of each planet on sky.

$$\text{Orbit of the sky} = 57753336 \times 12 \times 30 \times 60 \times 10 \text{ yojanas}$$

$$= 12474720576000 \text{ yojanas.}$$

$$\text{Orbit of the Sun} = 12474720576000 / 4320000$$

$$= 2887666 \text{ and } 4/5 \text{ yojanas.}$$

Similarly, orbit of other planets can be calculated by dividing the Orbit of the sky divided by number of rotations of the planet in a Yuga to get Orbit of the planet. Here yojana is the linear distance travelled by Object in their orbital path in sky.

Next, The orbit of the constellations 60 times the orbit of the Sun. So,

$$\text{Orbit of the constellations (stars)} = 2887666.8 \times 60$$

$$= 173260008 \text{ yojanas.}$$

As mentioned above, in Indian astronomy, we have twelve signs in a Moon's revolution. They are 1. Maish or Mesa or Aries, 2. Vrish or Rishabham or Taurus, 3. Mithun or Mithunam or Gemini, 4. Kark or Katakam or Cancer, 5. Singh or Simma or Leo, 6. Kanya or Kanni or Virgo, 7. Tula or Tulam or Libra, 8. Vrishchik or Vruchigam or Scorpio, 9. Dhanu or Dhanusu or Sagittarius, 10. Makar or Makaram or Capricorn, 11. Kumbh or Kumbham or Aquarius, 12. Meen or Meenam or Pisces.

Constellations mentioned in Atharvaveda are Kṛttikā, Rohinī, Mrigashīrsha, Ārdrā, Punarvasu, Pushya, Ashleshā, Maghā, Purva phalguni, Uttara phalguni, Hasta, Chitrā, Svāti, Vishākhā, Anurādhā, Jyeshthā, Mūla, Purva ashadha, Uttara ashadha, Shravana, Dhanishta, Satabhishak, Purva Bhadrapada, Uttara Bhadrapada, Revati, Ashvini and Bharani.

Stanza 7:

नृषि योजनम्, जिला भू-व्यासो, ऽर्केन्द्रोर्घ्रिजा गिण, क मेरोस् ।

भृगु-गुरु-बुध-शनि-भौमास् शशि-ङ-ज-ण-न-मांशकास् समार्कसमास् ॥ १.७ ॥

“8000(नृ) make a yojanam(योजनम्). The diameter(व्यासो) of the Earth(भू) is 1050(जिला) yojanas; of the Sun and the Moon, 4410(र्घ्रिजा) and 315(गिण) yojanas, (respectively); of Meru(मेरोस्), 1(क) yojana; of Venus(भृगु), Jupiter(गुरु), Mercury(बुध), Saturn(शनि) and Mars(भौमास्), one-fifth, one-tenth, one-fifteenth, one-twentieth and one-twentyfifth(respectively), of the Moon's diameter. The years are solar years”

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1 yojana is 8000. Here, 8000 refers to 8000 dhanus. नृ also refers to nara, purusa, dhanu and danda.

Linear diameter in yojanas for the Earth, the Sun and the Moon are 1050, 4410 & 315 yojanas respectively. Diameter of Meru is 1 yojana.

Linear diameters in yojanas at Moon's mean distance for Mars, Mercury, Jupiter, Venus & Saturn is 12.6(315/25), 21(315/15), 31.5(315/10), 63(315/5), 15.75(315/20) respectively.

We will use these values later to define the diameters and distances in modern measurement reference to compare the accuracy of data shared by Aryabhata.

Stanza 8:

भापक्रमो स्प्रहांशा शशिविक्षेपऽपमण्डलात् झ अर्धम् ।
शनि-गुरु-कुज ख-क-ग अर्ध, भृगु-बुध ख, स्वाङ्गुलस् घहस्तस् ना ॥ १.८ ॥

"The greatest declination of the Sun is 24deg. The greatest celestial deviation from the ecliptic of the Moon is 4 ½ deg; of Saturn, Jupiter and Mars, 2deg, 1deg and 1 ½ deg respectively; and of Mercury and Venus, 2deg. 96 angulas(अङ्गुल) or 4 cubits (घहस्तस्) make a नृ or नृ"

We will take the last point as it is related to the last stanza.

96 angulas make 4 cubits. i.e., 24 angulas makes 1 cubit.

96 angulas make 1 nr & 8000 nr make 1 yojana.

Since, Earth's diameter is 1050 yojanas, according to modern value of diameter of Earth is 12757 km. From this we can convert the above data in modern reference.

1 yojana = 12.1111 km.

1 nr = 152 cm.

1 cubit = 38 cm which is 1.25 feet.

The greatest declination of the Sun is the obliquity of the ecliptic. Value given by Aryabhata is 24deg. Modern value is between 23deg 27min to 23deg 47min measured in Julian centuries from 1900AD. Value in common use is 23.5deg.

Inclination of Orbits of Moon is 4.5deg; Mars, 1.5deg; Mercury, 2deg(geocentric); Jupiter, 1deg; Venus, 2deg(geocentric); Saturn 2deg.

Modern values of inclination of Orbits of Moon is 5.15deg; Mars, 1.85deg; Mercury, 7deg(heliocentric); Jupiter, 1.2deg; Venus, 3.4deg(heliocentric); Saturn 2.5deg.

Stanza 9:

बुध-भृगु-कुज-गुरु-शनि न-व-रा-ष-ह गत्वा अंशकान् प्रथमपातास् ।
सवितुरमीषां च तथा द्वा-जखि-सा-हदा-हल्य-खिच्य मन्दौच्चम् ॥ १.९ ॥

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“The ascending nodes of Mercury(बुध), Venus(भृगु), Mars(कुज), Jupiter(गुरु) and Saturn(शनि) having moved to 20deg(न), 60deg(व), 40deg(रा), 80deg(ष), 100deg(ह) respectively; and the apogees of the Sun and the same planets having moved to 78deg(द्वा), 210deg(जखि), 90deg(सा), 118deg(हदा), 180deg(हल्य) and 236(खिच्य) respectively”

Above stanza gives the longitudes of the ascending nodes and the apogees of the planets for AD499 given by Aryabhata.

Longitudes of the Ascending nodes for Mars, Mercury, Jupiter, Venus and Saturn are 40deg, 20deg, 80deg, 60deg and 100deg respectively. Modern values by calculation are 37deg 49min, 30deg 35min, 85deg 13min, 63deg, 16min and 100deg 32min.

Longitudes of the Apogees for Sun, Mars, Mercury, Jupiter, Venus and Saturn are 78deg, 118deg, 210deg, 180deg, 90deg and 236deg respectively. Modern calculated values are 77deg 15min, 128deg 28min, 234deg 11min, 170deg 22min, 290deg 4min and 243deg 40min respectively.

In the above stanza, *gatva* means “having moved” which tells that Aryabhata believed in planet moving around sun. Bhaskra (pupil of Aryabhata), in his commentary, says that the ascending node and apogee data are shared to Aryabhata and he remembered the revolution numbers by continuing of tradition. This means that this information shared are older than Aryabhata himself and is shared to them as part of their teachings. It is also fact that Joshiya which is part of veda teachings also has these data in decrypted form.

Stanza 10:

झ अर्धानि मन्दवृत्तं शशिन स्त्र, ग छ घ ढ छ झ यथाउक्तेभ्यस् ।
झा-ग्द-ग्ला-र्ध-द्द तथा शनि-गुरु-कुज-भृगु-बुधौचशीघ्रेभ्यस् ॥ १.१० ॥

“The manda(मन्दा) epicycles of the Moon, the Sun, Mercury, Venus, Mars, Jupiter and Saturn are, respectively, 7(छ), 3(ग), 7(छ), 4(घ), 14(ढ), 7(छ) and 9(झ) each multiplied by 4 ½ ; the sighra(शीघ्रे) epicycles of Saturn, Jupiter, Mars, Venus, Mercury are, respectively, 9(झा), 16(ग्द, 3+13), 53(ग्ला, 3+50), 59(र्ध, 19+40) and 31(द्द, 18+13) each multiplied by 4 ½”

Stanza 11:

मन्दात् ङ ख द ज डा वक्रिणां द्वितीये पदे चतुर्थे च ।
जा ण क्ल छल झ उच्चात्शीघ्रात् गियिङ्गश कुवायुकक्ष्याअन्या ॥ १.११ ॥

“The manda(मन्दा) epicycles of the retrograding planets in the second and fourth anomalistic quadrants are, respectively, 5(ङ), 2(ख), 18(द), 8(ज) and 13(डा) each multiplied by 4 ½ ; and the sighra(शीघ्रे) epicycles of Saturn, Jupiter, Mars, Venus and Mercury are, respectively, 8(जा), 15(ण), 51(क्ल, 1+50), 57(छल, 7+50) and 29(झ, 9+20) each multiplied by 4 ½. 3375(गियिङ्गश, 5+70+300+3000) is the outermost circumference of the terrestrial wind”

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Above two stanzas are on Manda and Sighra corrections involved in finding longitudinal position of planets. We will see in detail when we see on calculating position of planets. We will first see what is shared in these stanzas and we will what is Manda and Sighra.

Manda epicycles of planets- Odd quadrant: Sun - $13.5(3 \times 4.5)$ deg, Moon – $31.5(7 \times 4.5)$ deg, Mars – $63(14 \times 4.5)$ deg, Mercury – $31.5(7 \times 4.5)$ deg, Jupiter – $31.5(7 \times 4.5)$ deg, Venus – $18(4 \times 4.5)$ deg & Saturn – $40.5(9 \times 4.5)$ deg.

Sighra epicycles of planets- Odd quadrant: Mars – $238.50(53 \times 4.5)$ deg, Mercury- $139.50(31 \times 4.5)$ deg, Jupiter- $72(16 \times 4.5)$ deg, Venus- $265.50(59 \times 4.5)$ deg & Saturn- $40.50(9 \times 4.5)$ deg.

Manda epicycles of planets – Even quadrant: Sun – $13.5(3 \times 4.5)$ deg, Moon- $31.5(7 \times 4.5)$ deg, Mars- $63(14 \times 4.5)$ deg, Mercury- $31.5(7 \times 4.5)$ deg, Jupiter- $31.5(7 \times 4.5)$ deg, Venus- $18(4 \times 4.5)$ deg & Saturn- $40.5(9 \times 4.5)$ deg.

Sighra epicycles of planets- Even quadrant: Mars- $229.50(51 \times 4.5)$ deg, Mercury- $130.50(29 \times 4.5)$ deg, Jupiter- $67.50(15 \times 4.5)$ deg, Venus- $256.50(57 \times 4.5)$ deg & Saturn – $36(8 \times 4.5)$ deg.

We will see concept of Manda and Sighra epicycle and their role in finding position of planets in next post. We will see in details in relevant chapters after completing the trigonometric table (sine tables,etc..) given by Aryabhata and then Ganita pada (for calculation basics like multiplication, cube root, square root, etc..) for understanding further calculations.

Let us see the concept of Manda and Sighra epicycles. Consider Mercury, which is closer to Sun than Earth. It revolves around Sun and considering Geocentric concept, in our sky mercury also goes around us as Earth is also rotating about itself and about Sun. So, to find the position of Mercury with respect to Earth sky position reference, we need to two epicyclic paths, with which we can approximately denote the path of eccentric elliptical orbit of planets around Earth. However, there are other corrections also involved. To understand the basic concept for now, we will keep it to this level. As mentioned in my earlier post, we will see the calculation in details and finding the position of planets in later chapters. This post is only to explain the concept. Let me try to put in simple terms for easy understanding. Try to refer the image references given for each point and move onto the next point.

Refer image1 to understand the concept in simple terms. However, in our case planet position p will always be horizontal.

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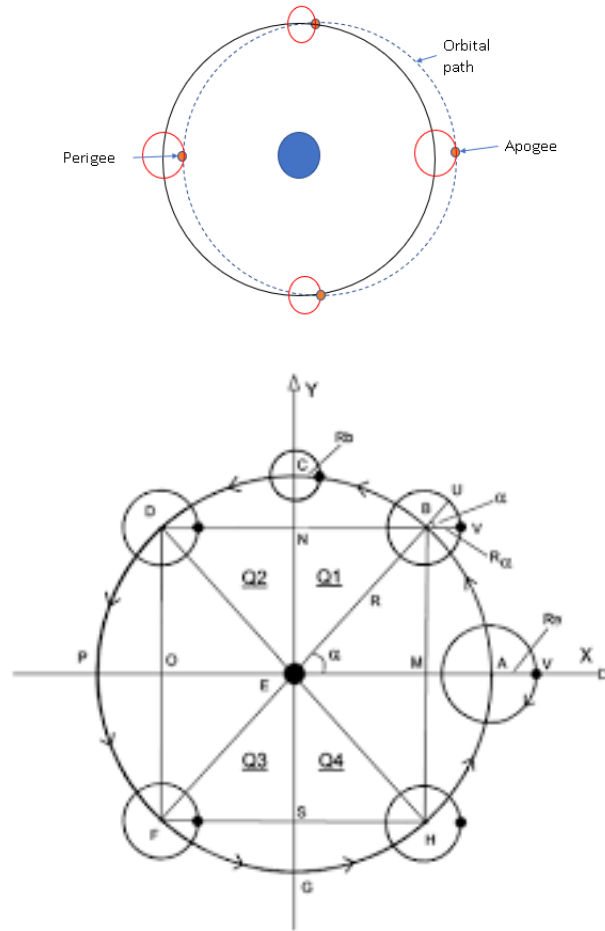


Image 2 shows the pulsating epicycle scheme in detail. The epicycle starts out from the apogee A (we have already seen that apogee is the farther most point of the planet in its elliptic path), where the planet is on the far side at V on the epicycle. At this point it is farthest from the Earth(E). As it moves counter clockwise, the epicycle's radius contracts till it reaches a minimum at the quadrature C (Rb is lesser as it contracts). The planet meanwhile is moving clockwise on the epicycle with an angular velocity that is equal to that of the epicycle on the deferent.

The center of the epicycle moves on the deferent whose parametric equations are:

$$X = R \cos (\alpha)$$

$$Y = R \sin (\alpha)$$

Where R is the radius of the deferent.

As explained earlier, in this scheme, the line joining the center of the epicycle to the planet will always remain parallel to the apsidal line or the X axis. Due to this, the Y coordinate of the planet will be identical with that of the epicycle center at all times. The X coordinate of the planet will be offset from that of the center of the epicycle by an amount that equals the radius of the epicycle. But note that unlike the eccentric case, the epicycle radius in the pulsating model is not a constant. It varies with position (or the anomaly angle α).

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Denoting the instantaneous radius of the epicycle at any arbitrary angle α as $R\alpha$, the planetary orbital parametric equation become:

$$X = R \cos(\alpha) + R\alpha$$

$$Y = R \sin(\alpha)$$

As mentioned, had $R\alpha$, been a constant (a constant-radius epicycle), the resultant planetary orbit would have been an eccentric circle. However, in the present case there is a pulsation, which decreases and increases the radius $R\alpha$. Now let us see method to find $R\alpha$.

Given a mean planet position, the first step in the process is the determination of the quadrant sequence number. This number indicates how many quadrants away from the apogee the mean planet is located. Having this found the quadrant, the next step is to determine the instantaneous epicycle radius at that point in the orbit, for which we are first required to establish the base sine.

Q1- Odd- Part past (arc AB) – Sin angle (AEB) – Sin (α)

Q2- Even – part to come (arc DP) – Sin angle (PED) – Sin (α)

Q3 – Odd – part past (arc PF) – Sin angle (FEP) – negative Sin (α)

Q4- Even- part to come (arc HA) – Sin angle (AEH) – negative Sin (α)

The final verses for determination of the corrected epicycle dimension are as follows:

For the single-epicycle scheme (ie. Those for the Sun and Moon), the even epicycle is always greater than the odd (ie $R_a > R_b$). Thus, the result will have to be subtracted from the even epicycle to obtain the corrected epicycle.

In other words,

$R\alpha = R_a - (R_a - R_b) \sin(\alpha)$ for quadrants Q1 & Q2; ie.,

$$= R_a - \text{del}(r) \sin(\alpha)$$

$R\alpha = R_a - (R_a - R_b) (-\sin(\alpha))$ for quadrants Q3 & Q4.

$$= R_a + \text{del}(r) \sin(\alpha)$$

Where $\text{del}(r)$ is the difference between the epicycles at the end of the even and odd quadrants.

Based on the $R\alpha$ equation above, we can find the equation for pulsating epicycle orbit of planets as

$$X = R \cos(\alpha) - \text{del}(r) \sin(\alpha) + R_a \text{ for quadrants Q1 \& Q2}$$

$$X = R \cos(\alpha) + \text{del}(r) \sin(\alpha) + R_a \text{ for quadrant Q3 \& Q4}$$

$Y = R \sin(\alpha)$ as it remains parallel to the apsidal axis X. With the above equations, we can calculate a planet's actual position in its orbit with respect to its mean anomaly. This post is only to show the importance of the next stanza. We will see above in detail

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later. However, how did Aryabhata found the sine of an angle to calculate as per above formula. Let us see that in next stanza.

2. Interesting thoughts

Light:

Light is a very important entity in many field of study. Light or visible light is visible to human beings. There are living being which can see waves other than visible light also. We see light and its colours because of some interesting works of our eyes.

We dont know many things what WE(our eyes) do. There are infinite number of colours available, like there are infinite numbers between two numbers. Our eyes do a remarkable job in identifying the difference between numerous colours. All these things happen because of light. As per evolution theory, If our body, what we have now is because of our changing needs and for our survival, then why do we need to see so many colours? But we need Energy.

Visible light falls under a range of wavelength which also has different colours in different wavelength. Combination of colour(waves of different wavelength) will lead to many other colours like white, purple, etc.,

Any object we see, are because of the light reflected by them. Our eye has a lens which detects light, controls the amount and focus the light onto a small portion called cone cells. These cells will identify the colour and gives signal to brain.

We know how light looks for human eyes. But what is this light?

Lights are wave of photons. Photons are small packets of energy. These packets travel at measurable speed which is very fast that as per Einstein, no particle can travel faster than light.

If objects can absorb some portion(wavelength) of light, may be because it is good for them or they need them, others are reflected back. So that means they actually take those packets of Energy and reflect some back. Objects can also generate light by releasing energy by heat and by chemical reaction. Whether the velocity of light is always universal? If something has enough attractive power to pull the light back then it means that emitted light is moving away from it, slowed down, stopped and then pulled back towards it. So can speed of light change if such object exists? If an object travels faster than light and moving away from us, then can we see them? If such object exists then we will observe like they disappeared as we cant see the light emitted or reflected by them. Light if travels as a whole in speed of light and photons moved in wave form then can photon which need to travel is wave form is moving faster than speed of light? Can photons collide with each other? What happens when they collide? Can medium on which light travels slow down their speed?

Light, energy packets, are source of any transaction among objects. There are living being which absorb more energy from Sun light than the other. Black people absorb more energy from Sun. They are comparatively more healthier than White shin people. Hence, white skin is a deficiency as they are unable to absorb enough energy from Sun light. There are some plants which feed only on light like Spirulina. It takes more energy from Sun compared to other plants. Coming to bigger picture, any chemical elements and its properties depend on the electrons in its and interaction of electrons between other elements. Photons can free an electron to make them move around and electrons can also reverse by releasing photons. Such

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small packets of energy has potency in creation of universe, has tejas, has enough strength to move in its path and to control anything in the universe. Can photons be the means by which Brahman controls the Universe?

Can there be something within photon which is doing this? Such a beautiful web of so many entities connected by these packets of energy passing between them. Science named it Universe.

Light, ultimately drives many basic principles in life structure, medicine, physics, chemistry and anything which we see and we can't see. So, everything what we see is controlled by the Supreme Brahman and everything that exists are part of our beautiful Universe which is the body of supreme Brahman. Answer to the question, can we see Brahman? We can see him everywhere if we open our eyes.

Stanza 12:

मखि भखि फखि धखि णखि जखि ङखि हस्झ स्क्कि किष्ण श्घकि किघ ।

घ्लकि किग्र हक्य धकि किच सा झश ङ्व क्ल प्त फ छ कलार्धज्यास् ॥ १.१२ ॥

“225(मखि, 25+200), 224(भखि, 24+200), 222(फखि, 22+200), 219(धखि, 19+200), 215(णखि, 15+200), 210(जखि, 10+200), 205(ङखि, 5+200), 199(हस्झ, 100+90+9), 191(स्क्कि, 90+1+100), 183(किष्ण, 100+80+3), 174(श्घकि, 70+4+100), 164(किघ, 100+4+60), 154(घ्लकि, 4+50+100), 143(किग्र, 100+40+3), 131(हक्य, 100+1+30), 119(धकि, 19+100), 106(किच, 100+6), 93(सा, 90+3), 79(झश, 9+70), 65(ङ्व, 5+60), 51(क्ल, 1+50), 37(प्त, 21+16), 22(फ, 22) & 7(छ, 7) - these are the Rsine-differences in terms of minutes of arc”

Aryabhata is the first person in history to give Rsine values. He has also shared method of calculating the same in later chapters.

Here there are 24 values of Rsine-differences. Considering these values are for 90deg, then each value is given for the interval of $90/24=3.75\text{deg}$, which is 225min(matching the first value).

Rsine values can be calculated by the adding with the next as 225, 449, 671, 890, 1105, 1315, 1520, 1719, 1910, 2093, 2257, 2431, 2585, 2728, 2859, 2978, 3084, 3177, 3256, 3321, 3372, 3409, 3431, 3438.

Rsine values with relevant angles (given inside bracket) are as 225(3.75deg), 449(7.5deg), 671(11.25deg), 890(15deg), 1105(18.75deg), 1315(22.5deg), 1520(26.25deg), 1719(30deg), 1910(33.75deg), 2093(37.5deg), 2257(41.25deg), 2431(45deg), 2585(48.75deg), 2728(52.5deg), 2859(56.25deg), 2978(60deg), 3084(63.75deg), 3177(67.5deg), 3256(71.25deg), 3321(75deg), 3372(78.75deg), 3409(82.5deg), 3431(86.25deg), 3438(90deg).

We know that Sin(90deg) is one so dividing a factor of 3438 in above values will give Sine values. Let us compare the results. Modern value is mentioned inside brackets.

Sin(0) = 0(0); Sin(3.75) = $225/3438=0.0654(0.0654)$; Sin(7.5) = $0.1305(0.1305)$; Sin(11.25) = $0.1951(0.19509)$; Sin(15) = $0.2588(0.2588)$; Sin(18.75) = $0.3214(0.3214)$; Sin(22.5) = $0.3824(0.3826)$; Sin(26.25) = $0.4421(0.4422)$; Sin(30) = $0.5(0.5)$; Sin(33.75) = $0.5555(0.5555)$; Sin(37.5) = $0.6087(0.6087)$; Sin(41.25) =

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$0.6564(0.6593)$; $\sin(45) = 0.7070(0.7071)$; $\sin(48.75) = 0.7518(0.7518)$; $\sin(52.5) = 0.7934(0.7933)$; $\sin(56.25) = 0.8315(0.8314)$; $\sin(60) = 0.8662(0.8660)$; $\sin(63.75) = 0.8970(0.8968)$; $\sin(67.5) = 0.9240(0.9238)$; $\sin(71.25) = 0.9470(0.9469)$; $\sin(75) = 0.9659(0.9659)$; $\sin(78.75) = 0.9808(0.9807)$; $\sin(82.5) = 0.9915(0.9914)$; $\sin(86.25) = 0.9979(0.9978)$; $\sin(90) = 1(1)$.

It is interesting to see how these values are given by Aryabhata in terms of Whole numbers instead of sharing in decimal places.

Stanza 13:

दशगीतिकसूत्रमिदं भूग्रहचरितं भपञ्जरे ज्ञात्वा।
ग्रहभगणपरिभ्रमणं स याति भित्त्वा परं ब्रह्म ॥ १.१३ ॥

“Knowing this Dasagithika Sutra, the motion of the Earth and the planets, on the Celestial Sphere, one attains the Supreme Brahman after piercing through the orbits of the planets and stars”

Here Dasagithika refers to 2 to 11 stanzas. First is Mangalacharanam & 12 is on Sine table which are approximation values.

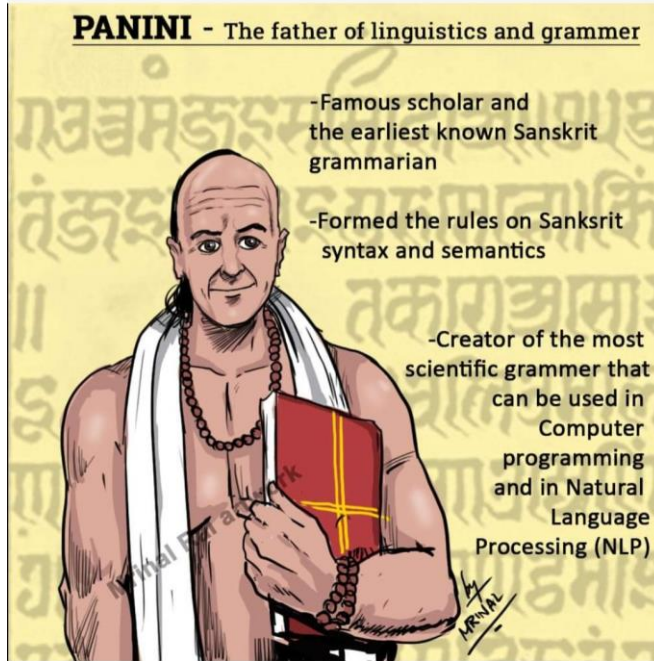
In this stanza, Aryabhata refers to the path of Moksha (reaching the Supreme Brahman) which is after piercing through the orbits of the Planets and stars. As per Sri Mahabharata verses santi 254.18,19,20 addressing the Supreme Brahman states that after exhaustion of punya and papa, those who start their journey to Vishnu loka pass through Sun to attain the feet of Lord Sriman Narayana. There are verses in Upanishads also on this referring the path is through Orbits of planets and through Sun.

With this Chapter III, Githika pada is over.

A Brief on ARYABHATIYA

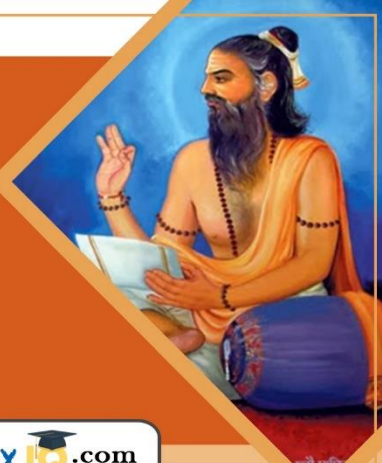
8. Do you know?

PANINI - The father of linguistics and grammar



9. Do you know?

Panini Rishi

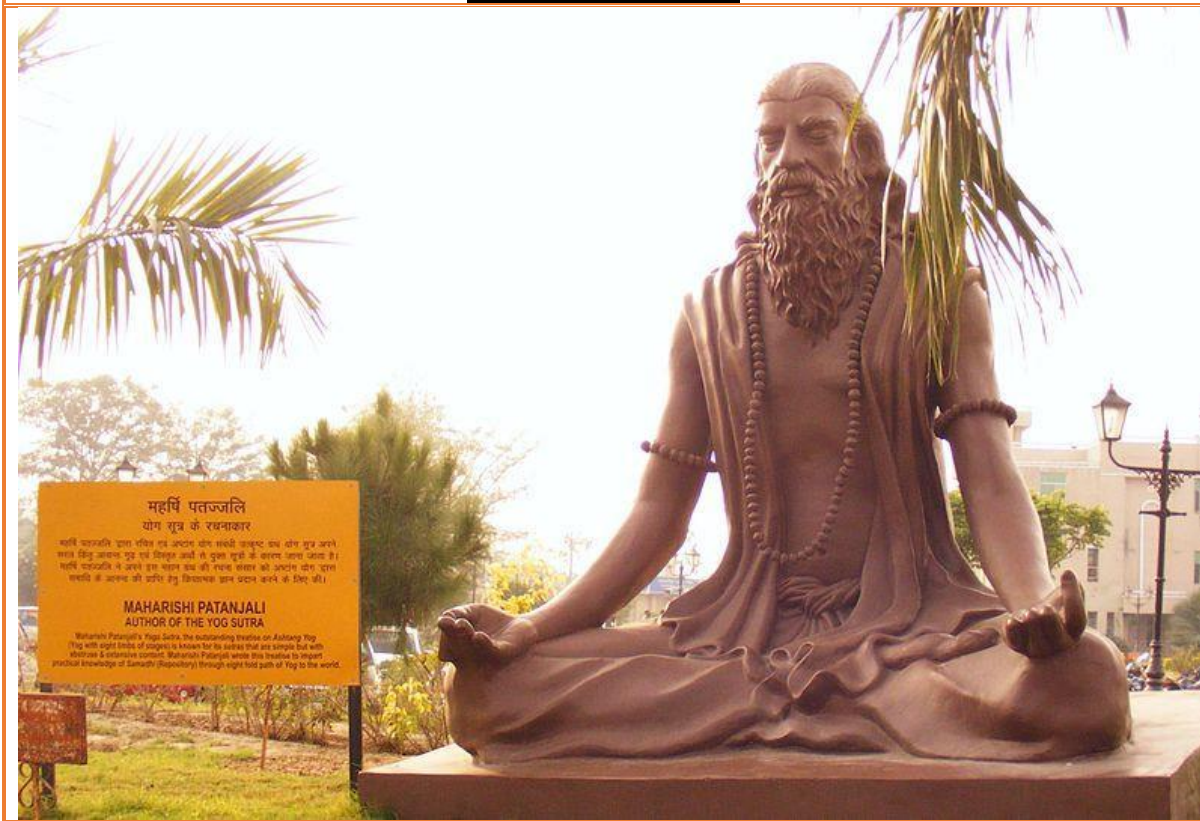


www.BiographyIQ.com

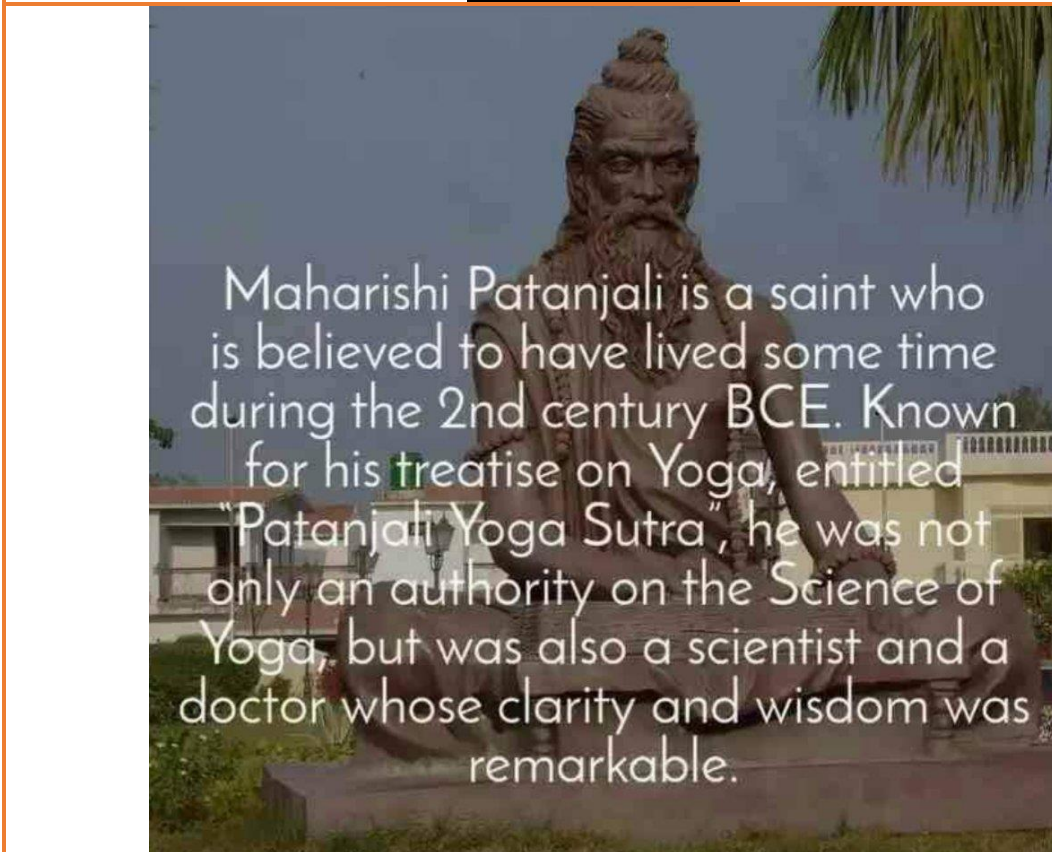
- Panini (4th century BCE or "6th to 5th century BCE") was an ancient grammarian, and a revered scholar in ancient India. Considered the father of linguistics, Panini likely lived in the northwest Indian sub-continent during the Mahajanapada era.
- Pānini is known for his text Ashtadhyayi, a sutra-style treatise on Sanskrit grammar, 3,959 "verses" or rules on linguistics syntax and semantics "eight chapters" which is the foundational text of the Vyākaraṇa branch of the Vedāṅga.
- His formalization of language seems to have been influential in the formalization of dance and music by Bharata Muni. His ideas influenced and attracted commentaries from scholars of other Indian religions such as Buddhism.

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10. Do you know?



11. Do you know?



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Chapter IV. Ganita Pada

Stanza 1:

ब्रह्म-कु-शशि-बुध-भृगु-रवि-कुज-गुरु-कोण-भगणान् नमस्कृत्य ।
आर्यभटसितिविह निगदति कुसुमपुरे अभ्यर्चितं ज्ञानम् ॥ २.१ ॥

“Having bowed with reverence to Supreme Brahman, Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn and the Asterisms, Aryabhata sets forth here the knowledge honoured at Kusumapura”

Kusumapura is Patliputra (Patna).

Stanza 2:

एकं दश च शतं च सहस्रं अयुत नियुते तथा प्रयुतम् ।

कोटि अर्बुदं च वृन्दं स्थानात्स्थानं दशगुणं स्यात् ॥ २.२ ॥

“Eka (एकं), dasa (दश), sata (शतं), sahasra (सहस्रं), ayuta (अयुत), niyuta (नियुते), prayuta (प्रयुतम्), koti (कोटि), arbuda (अर्बुदं), and vrnda (वृन्दं) are, respectively, from place to place, each ten times (दशगुणं) the preceding”

That means, Eka is units place, dasa is tens place, sata is hundreds place, sahasra is thousands place, ayuta is ten thousands place, niyuta is hundred thousands place, prayuta is millions place, koti is ten millions place, arbuda is hundred millions place, and vrnda is thousand millions place. This is simple, normal way of representing power of number when we write. As Einstein once said counting numbers started from India.

Stanza 3:

वर्गस्समचतुरश्रस्फलं च सदृश द्वयस्य संवर्गस् ।

सदृश त्रयसंवर्गस्थनस्तथा द्वादशाश्रिस् स्यात् ॥ २.३ ॥

“An equilateral quadrilateral with equal diagonals and also the area thereof are called “Square”. The product of two equal quantities is also square. The continued product of three equals as also the solid having twelve edges is called a “Cube””

“Samacaturasra” indicates that four sided figure whose sides are equal to one another and whose diagonals are also equal to each other is called a samacaturasra.

By defining a square as the product of two equal quantities Aryabhata has stated the rule of squaring. That is, to find the square of a number, one should multiply (संवर्गस्) that number by itself.

The rule of cubing a number is implied as in the squaring case which is continued product of three equal quantities as in Solid having 12 edges “Cube”.

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Stanza 4:

भागं हरेदवर्गान्नित्यं द्विगुणेन वर्गमूलेन ।
वर्गाद्धिर्गो शुद्धे लब्धं स्थानान्तरे मूलम् ॥ २.४ ॥

“Always divide the even place by twice the square root. Then having subtracted the square from the odd place, set down the quotient at the next place. This is the Square root.”

We will take an example, Square root of 55225.

Odd places can be subtracted by greatest possible square closer to it & then for even places by twice the square root. Odd and even places are below:

o e o e o

5 5 2 2 5

We will represent line of square root(answer) as LSR & Dividing number as DN. First odd place number is 5(o), closest greatest square is 4(square of 2). Subtract 4 in 5 and write square root of 4, i.e. 2 in LSR and remainder as 1.

Reminder with even number, which 1 & 5(e) makes 15. Divide this by the twice the LSR, i.e., $15/(2 \times 2) = 15/4 =$ quotient 3 remainder 3. Write quotient in LSR next to the number present (2) as 23.

Reminder with odd number, which is 3 & 2(o) makes 32. Now subtract the square of the last quotient as $32 - 3 \times 3 = 32 - 9 = 23$. Place the next even number (2) with this as 232. Now divide by twice the LSR i.e., $232/(23 \times 2) = 232/46 =$ quotient 5 remainder 2. Write quotient in LSR next to the number present (23) as 235.

Reminder with odd number, which is 2 & 5(o) makes 25. Subtract the square of the quotient, $25 - 5 \times 5 = 25 - 25 = 0$.

Process ends. The square root is the number available in LSR which is 235. So square root of 55225 is 235. Reminder is zero so exact square.

This applies even to fraction square roots. If let us say the number is 55226 instead of 55225. Then reminder will be 1 at the end then next process is to take twice the LSR which is 470 (235×2) and divide it in reminder as we did before. That will give $1/470$ as fraction. So Square root of 55226 is $235 \frac{1}{470}$.

This also means that there are 470 unit place numbers in between the next square number from 55225. Square of 236 is 55696. $55696 (236 \times 236) - 55225 (235 \times 235) = 471$.

236×2 makes 472. So, there are 472 unit place numbers available between 236×236 & 237×237 . i.e., $56169 (237 \times 237) - 55696 (236 \times 236) = 473$. Now you can see the relation in square numbers. Try and find relation for square of even set of numbers like 22, 56, 2345, etc.,.

This is the logic used by Aryabhata and we are following the same logic even now in our school mathematics.

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Stanza 5:

अघनात् भजेत् द्वितीयात् त्रिगुणेन घनस्य मूलवर्गेण ।
वर्गस् त्रिपूर्वगुणितस्शोध्यस् प्रथमात्घनस्य घनात् ॥ २.५ ॥

“Divide the second non-cube place by thrice the square of the cube root; subtract from the first non-cube place the square of the quotient multiplied by thrice the previous and the cube from the cube place”

First let us see what is cube place and non-cube place. Beginning from the unit place, cube places are notated as ‘c’, first non-cube as ‘n’ & second non-cube as ‘n’.

Number is to be coded as

cn'ncn'ncn'ncn'nc.

Let us take an example: Cube root of 1771561

Denote the places as 1(c),7(n'),7(n),1(c),5(n'),6(n),1(c)

First subtract the greatest close cube with cube place, i.e., 1(c) – cube of 1 = remainder 0. LSR is 1.

Then divide the second non-cube by thrice the square of the cube root, i.e., divide 07 by $3 \times (\text{square of } (1)(\text{LSR})) = 7/3 = \text{quotient } 2 \text{ remainder } 1$. Write quotient in LSR as 12.

Combine remainder with next number (first non-cube number) as 17. Subtract from the first non-cube number place the square of the quotient multiplied by thrice the previous, i.e., Subtract $3 \times 1 \times \text{square of } (2)$ from $17 = 17 - 3 \times 1 \times 4 = 17 - 12 = \text{remainder } 5$.

Combine the remainder with cube place number as 51. Now continue the cycle.

$51 - 8 (\text{cube of } (2)) = \text{remainder } 43$.

Combine with next, 435.

Divide by $3 \times \text{square of } (12)$ as $435 / (3 \times 144) = 435 / 432 = \text{quotient } 1 \text{ remainder } 3$. LSR = 121.

Combine with next number as 36.

Subtract $3 \times 12 \times \text{square of } (1)$, $36 - 36 = \text{remainder } 0$.

Combine with next number as 01.

Subtract cube of (1) as $1 - 1 = \text{remainder } 0$.

So, cube root of 1771561 is 121. Exact cube. Let us take two more example for fraction.

Second example: Cube root of 12977875

Denote the places as 1(n),2(c),9(n'),7(n),7(c),8(n'),7(n),5(c).

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First subtract the greatest close cube till the cube place, i.e., $12 - 8$ (cube of (2)) = remainder 4. LSR is 2.

Combine the remainder with next number as 49.

Then divide the second non-cube by thrice the square of the cube root, i.e., divide 49 by $3 \times$ (square of (2) (LSR)) = $49/12$ = quotient 3 remainder 13. Here, note that we have kept 13 as remainder, this is done to avoid negative number as remainder in further steps. Write quotient in LSR as 23.

Combine remainder with next number (first non-cube number) as 137. Subtract from the first non-cube number place the square of the quotient multiplied by thrice the previous, i.e., Subtract $3 \times 2 \times$ square of (3) from 137 = $137 - 3 \times 2 \times 9 = 137 - 54 =$ remainder 83. Note that if in previous step, if we have taken 1 as remainder then we have to subtract 54 in 17 which will give negative number as remainder.

Combine the remainder with cube place number as 837. Now continue the cycle.

$837 - 27$ (cube of (3)) = remainder 810.

Combine with next, 8108.

Divide by $3 \times$ square of (23) as $8108 / (3 \times 529) = 8108/1587$ = quotient 5 remainder 173. LSR = 235.

Combine with next number as 1737.

Subtract $3 \times 23 \times$ square of (5), $1737 - 1725 =$ remainder 12.

Combine with next number as 125.

Subtract cube of (5) as $125 - 125 =$ remainder 0.

So, cube root of 12977875 is 235. Exact cube. Let us take an example for fraction.

Third example: Cube root of 12977885.

I have taken a number with ten added to our earlier example. So, we will repeat few steps after LSR as 235 to see the difference in finding the answer. I have underlined the difference.

Divide by $3 \times$ square of (23) as $8108 / (3 \times 529) = 8108/1587$ = quotient 5 remainder 173. LSR = 235.

Combine with next number as 1738.

Subtract $3 \times 23 \times$ square of (5), $1738 - 1725 =$ remainder 13.

Combine with next number as 135.

Subtract cube of (5) as $135 - 125 =$ remainder 10.

We have a remainder so it is not an exact cube. So, to find the fractional portion. Go for the next step, which for second non-cube number, i.e., divide the second non-cube by

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thrice the square of the cube root, which is $3 \times \text{square of } (235) = 165675$. So the fractional part is $10/165675$.

So, cube root of 12977885 is $235\frac{10}{165675}$. Remember fractions are always better than approximate values represented in decimal places.

Stanza 6:

त्रिभुजस्य फलशरीरं समदलकोटीभुजा अर्धसंवर्गस् ।
ऊर्ध्वभुजातदसंवर्ग अर्ध सस्यनस् षषश्रिसिति ॥ २.६ ॥

“The product of the perpendicular (dropped from the vertex on the base, समदलकोटी) and half the base gives the measure of the area of a triangle(फलशरीरं). Half (not correct, should be one third) the product of that area (of the triangular base) and the height is the volume of a six-edged solid”

Second half translation provided is not exact as there are no commentary available on the second half by Pupils of Aryabhata, Bhaskara, Somesvara and Suryadeva as it may have some error by Aryabhata or not captured properly in written form. However, I have shared Volume of cone or pyramid with Brahmagupta shared.

First line is on Area of a triangle. समदलकोटी (Samadalakoti) means “perpendicular dropped from the vertex on the base of a triangle” (altitude of a triangle). फलशरीरं (phalaśārīraṁ) means “the measure or amount of the area”.

Area of a triangle = Altitude of a triangle (h) $\times$ (1/2) $\times$ base of a triangle (b)
 $= (1/2) \times b \times h$ (a known to us)

From this formula, other forms of the above formula like all sides (a, b, c) known, two sides and one angle known, etc., can be derived.

Second line, Volume of cone or pyramid as given by Brahmagupta is

Volume of cone or pyramid = $(1/3) \times \text{Area of base} \times \text{height}$.

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12. Do you know?

Electricity

Electroplating

कृत्रिमस्वर्णरजतलेपः संस्कृतिरुच्यते।

यवक्षारमयोधानौ सुशक्तजलमनिधौ॥

आच्छादयति तत्ताम्रं स्वर्णेन रजतेन वा।

सुवर्णलिप्तं तत्तस्मिन् शतकुम्भमिति स्मृतम्॥ (अगस्त्य संहिता)

ताम्रं यत्र पर कृत्रिम स्वर्णं वा रजत लेपं तेजावधानी धातु के घोल द्वारा बैटरी की सहायता से किया जाता है। जब ताम्र पूर्णरूप से स्वर्णलिप्त हो जाता है, तब उसे शतकुम्भ कहा जाता है।

Artificial Gold or Silver plating on copper plate can be done by using this battery. When plated with gold it is called as ShatKumbh.

अनेन जलभंगोस्ति प्राणो दानेषु वायुषु।

एवं शतानां कुम्भानां संयोगः कार्यकृत्स्मृतः॥

Agasti decomposed water by using his battery and observed that Pran(O₂) and Udan (H₂) were produced in ratio of 1:2 by volume

वायुबन्धकवस्त्रेण निबद्धो यानमस्तके

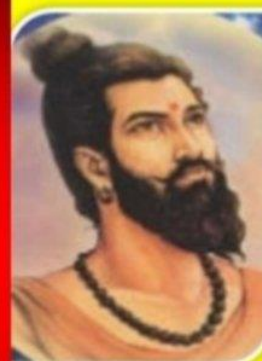
उदानः स्वतन्त्रत्वे विभर्त्याकाशचानकम्॥ (अगस्त्य संहिता शिल्प शास्त्र सार)

Udan was filled in balloon to fly it in air and Chanakya recommended to use it to escape from an enemy camp.

12. Do you know?

- He got the name kanad ,because
- *he was interested in minute particles called "kana".*
- His atomic theory can be a match to any modern atomic theory.

Kanad:

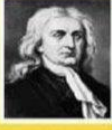


Rishi
KANADA
"VYSESHIKA
DARSHANAM"

Vedic Nuclear
Scientist

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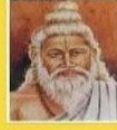
13. Do you know?



16th Century AD

Laws of motion

Newton vs Rishi Kanad



6th or 2nd Century BC

1) First Law of Motion -

Kanad : वेगः निमित्तविशेषात् कर्मणो जायते

Newton: The change of motion is due to impressed force

2) Second Law of Motion -

Kanad : वेग निमित्तापेक्षात् कर्मणो जायते नियन्दिक् क्रिया प्रबंध हेतु

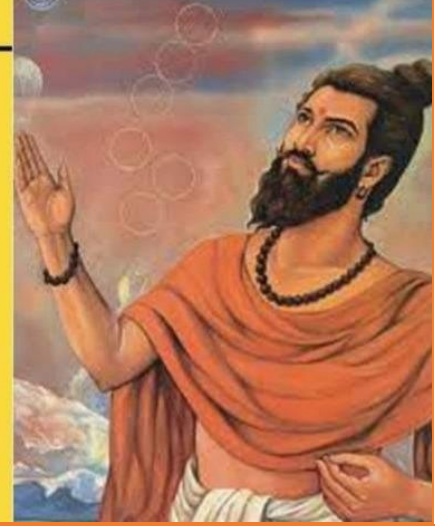
Newton: The change of motion is proportional to the motive force impressed and is made in the direction of the right line in which the force is impressed

3) Third Law of Motion -

Kanad : वेगः संयोगविशेषाविरोधी

Newton: To every action there is always an equal and opposite reaction

गति के नियम महर्षि कणाद ने दिए
ना की कोई न्यूटन फ्यूटन ने



14. Do you know?

CHARAK:



*Charak is called
the father of ayurvedic medicine
His Charak Samhita is a
remarkable book on
medicine.*

*Don't you find it fascinating
that thousands of years
back, medical science
was at such an advanced
stage in India.*

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Stanza 7:

समपरिणाहस्य अर्धं विष्कम्भ अर्धहतं एव वृत्तफलम् ।
तद्विजमूलेन हतं घनगोलफलं निरवशेषम् ॥ २.७ ॥

“Half of the circumference, multiplied by the semi-diameter certainly gives the area of a circle. That area multiplied by its own square root gives the exact volume of a sphere”

In this stanza, area of circle and volume of sphere formulae are shared.

Area of circle is Half the circumference multiplied by the half diameter (radius) gives the area of a circle. i.e.

Area of circle = $(1/2) \times \text{circumference} \times \text{radius}$

Also, as circumference calculation involves pi value, Aryabhata has shared a reference circle values to calculate approximate pi value. Eventhough, value upto 32 decimal and series formulae are available in Upanishad and Katapayadi system, Aryabhata shared an approximate value with whole numbers which can be used for our calculations.

Volume of sphere shared here is not exact as mentioned. However, can be applied if any equivalent volume of cube with area of face is known. This can be similar to finite element approximation technique which we will see in plane figures formula.

Bhaskara II shared exact and approximate formula for volume of sphere as below

Accurate formula,

Volume of Sphere = $(1/6) \times \text{surface area} \times \text{diameter}$

Approximate formula,

Volume of sphere = $(\text{diameter} \times \text{diameter}) \times (1/2) \times (1 + 1/21)$. In this formula value of pi is considered as 22/7.

Stanza 8:

आयामगुणे पार्श्वे तद्योगहते स्वपातलेखे ।
विस्तरयोग अर्धगुणे ज्ञेयं क्षेत्रफलं आयामे ॥ २.८ ॥

“Multiply the base and the face by the height, and divide by the sum of the base and the face; the results are the lengths of the perpendiculars on the base and the face. The results obtained by multiplying half the sum of the base and the face by the height is to be known as the area”

Here the stanza is on area of a Trapezium.

Let a be the bottom base length, b be the top face length & p be the height.

Term आयामगुणे means height of the trapezium. Term विस्तर means length & विस्तरयोग अर्धगुणे means half the sum of the base and the face.

Area of Trapezium = $(1/2) \times (\text{sum of base and face}) \times \text{height}$

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$$= (1/2) \times (a + b) \times p$$

Stanza 9:

सर्वेषां क्षेत्राणां प्रसाध्य पार्श्वे फलं तदभ्यासस् ।
परिधेस् षष्ठभागज्या विष्कम्भ अर्धेन सा तुल्या ॥ २.९ ॥

“In the case of all the plane figures, one should determine the adjacent sides (of rectangles after transformation) and find the area by taking their products. The chord of one-sixth of the circumference of a circle is equal to the radius”

First line is on finding the area of any plane figure, by dividing them into multiple rectangles and finding the area of those rectangle and adding them to get the area of plane figure or by converting the plane figure into equivalent rectangle and then calculating the area.

Second line is on chord length of one sixth of the circumference. If we divide a circle of 360deg circumference by 6. The result angle for the chord is $360/6 = 60\text{deg}$. Arms of the sector circle will be same as radius. Drawing a chord will make a triangle with chord, radius, radius as the sides of triangle. As the angle between the equal sides is 60degree. It will make an equilateral triangle. So, chord length will be same as radius.

Chord 60deg of circle = Radius

Which also, means $R \times \sin 30 = R/2$.

Stanza 10:

चतुरधिकं शतं अष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
अयुत द्वयविष्कम्भस्य आसन्नस्वृत्तपरिणाहस् ॥ २.१० ॥

“100 plus 4, multiplied by 8, and added to 62000. This is the nearly approximate measure of the circumference of a circle whose diameter is 20000”

This stanza gives approximate value of $\pi(\pi)$.

$(100 + 4) \times 8 + 62000 = 62832$. So, 62832 is approximate circumference of circle with diameter 20000.

We are using π (pi) to represent this factor which is given by Aryabhata as $62832/20000 = 3.1416$. This value does not occur in any earlier work (outside India) on Mathematics and forms an important contribution of Aryabhata. It is noteworthy that Aryabhata has called the above value approximate.

Some special notes from various other sources of π (pi) shared using Katapayadi code system in other slokas.

Karaṇa paddhati, written in the 15th century, has the following śloka for the value of pi (π)

അനുനൂനാനനനുനനിത്യ-

സ്ഥാഹതാശ്ചക്രകലാവിഭക്താഃ

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ചണ്ഡാംശുചന്ദ്രാധമകുന്ദിപാലൈർ-

വ്യാസസ്തദർദ്ധം ത്രിഭൗർവിക സ്യാത്

Transliteration

anūnanūnnānananunnānityai

ssmāhatāścakra kalāvibhaktōḥ

caṇḍāṃśucandrādhamakumbhipālair

vyāsastadarddham tribhamaurvika syāt

It gives the circumference of a circle of diameter, anūnanūnnānananunnānityai (10,000,000,000) as caṇḍāṃśucandrādhamakumbhipālair (31415926536).

Śaṅkara varman's Sadratnamālā uses the Kaṭapayādi system. A famous verse found in Sad-ratna-mālā is

भद्राम्बुद्धिसिद्धजन्मगणितश्रद्धा स्म यद् भूपगीः

bhadrāmbuddhisiddhajānmaṇitaśraddhā sma yad bhūpagīḥ

Splitting the consonants & giving values as per katapayadi code system, reversing the digits to modern-day usage of descending order of decimal places, we get 314159265358979324 which is the value of pi (π) to 17 decimal places, except the last digit might be rounded off to 4.

This verse encrypts the value of pi (π) up to 31 decimal places.

गोपीभाग्यमधुव्रात-श्रुङ्खिशोदधिसन्धिग ॥

खलजीवितखाताव गलहालारसंधर ॥

This verse directly yields the decimal equivalent of pi divided by 10: $\pi/10 = 0.31415926535897932384626433832792$

Surprising isn't? It will be more surprising if we really find the purpose of these terms shared with this much accuracy. I will come back to Aryabhatiya.

Stanza 11:

समवृत्तपरिधिपादं छिन्द्यात् त्रिभुजात् चतुर्भुजात् एव ।

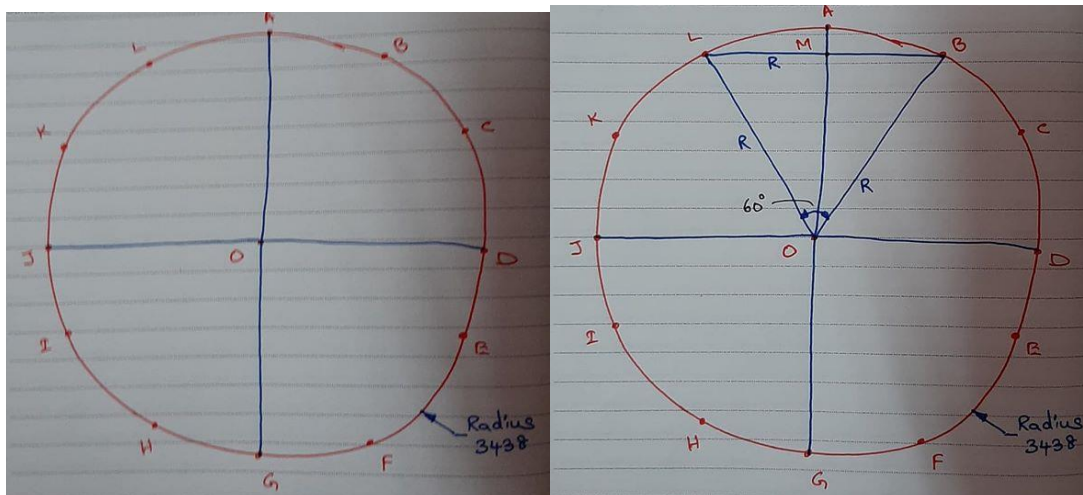
समचापज्या अर्धानि तु विष्कम्भ अर्धे यथा इष्टानि ॥ २.११ ॥

"Divide a quadrant of the circumference of a circle. Then from triangles and quadrilaterals, one can find as many Rsines of equal arcs as one likes, for any given radius"

This is about find Rsine values as mentioned in Gitika pada. We will take as example to for this. To compare the results with Gitika pada, we will take circle with radius 3438.

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We know that in Gitika pada (12th stanza), Aryabhata gave 24 Rsine values. This stanza explains the way for finding it. Method can be continued to any level if further smaller Rsine values are required.



First, divide a quadrant of the circumference of a circle. Let us divide into 12 divisions as in image. Then join LB to form a chord. Remember, in Ganita pada stanza 9, Aryabhata has shared Chord length of one-sixth of circumference which is equal to radius. LB is one-sixth of the circumference. So, we get first Rsine value. (Rsine in a right-angle triangle inscribed inside a circle with hypotenuse R is equal to opposite side of the angle)

$$1. \text{RSine}(\text{angle}(\text{BOM})) = \text{MB}$$

$$= \text{LB}/2$$

$$= 3438/2$$

$$= 1719$$

So, RSine30 = 1719.

$$2. \text{RSine}(\text{angle}(\text{MBO})) = \text{OM}$$

$$= \text{Sqrt}(\text{R}^2 - (\text{R}/2)^2)$$

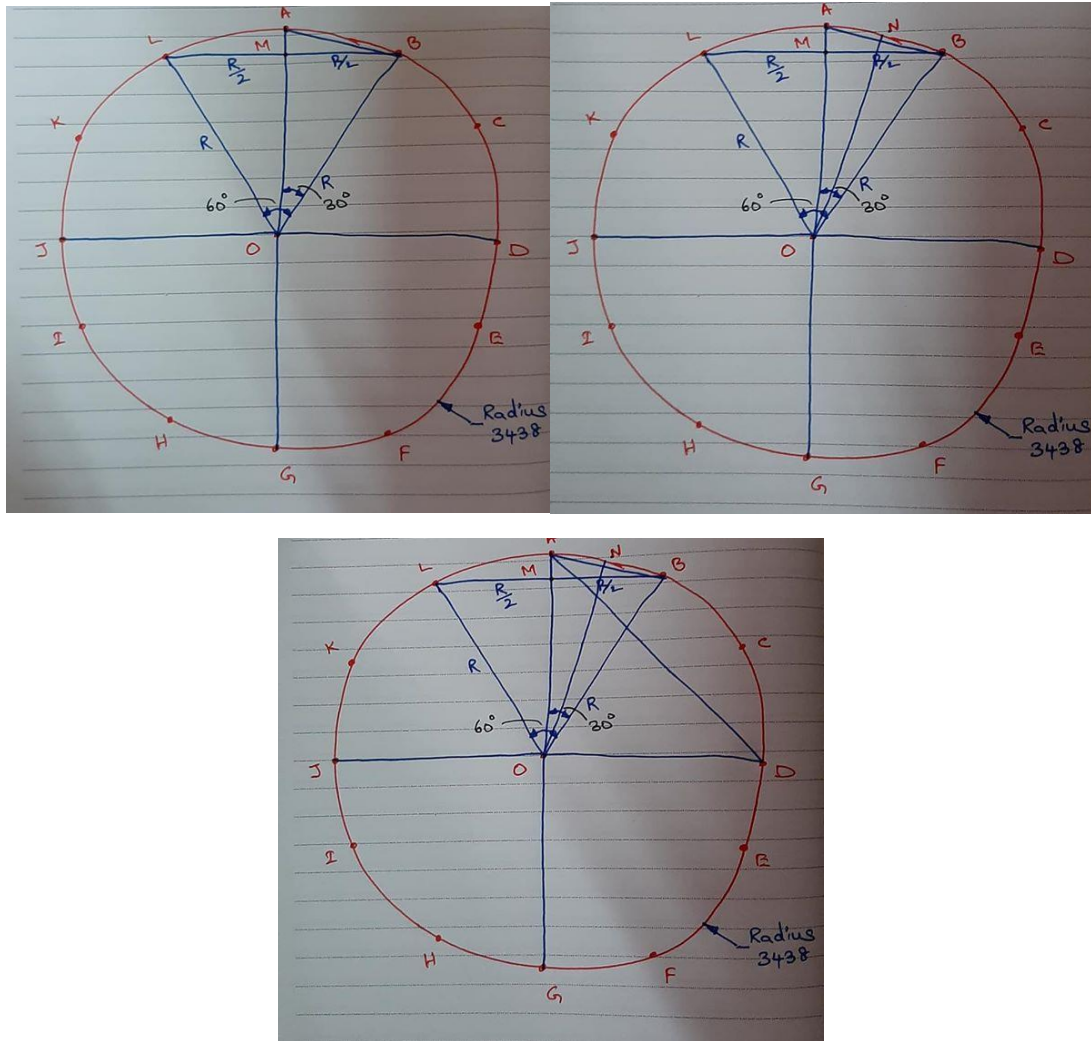
$$= \text{R} \times \text{sqrt}(1 - 1/4)$$

$$= \text{sqrt}(3) \times \text{R}/2$$

$$= 2978 \text{ (approx..)}$$

So, RSine60 = 2978.

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3. Refer image, generate a chord for 30deg which is AB. N is the midpoint of AB.

So, Chord 30 = $R \sin 15 = AB/2$ or AN.

$$AN = (\sqrt{AM^2 + MB^2})/2$$

$$AM = OA - OM$$

$$= 3438 - 2978$$

$$= 460 \text{ (approx.)}$$

$$MB = 1719$$

$$AN = (\sqrt{460 \times 460 + 1719 \times 1719})/2$$

$$= 890 \text{ (approx.)}$$

$$\text{So, } R \sin 15 = 890.$$

$$4. R \sin(\text{angle}(OAN)) = R \sin 75 = ON$$

$$ON = \sqrt{(OA^2 - AN^2)}$$

$$= \sqrt{3438 \times 3438 - 890 \times 890}$$

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$$= 3321 \text{ (approx..)}$$

5. Construct semisquare OAD. Here, Chord 90 is AD. Refer image.

$$\text{Chord } 90 = R\text{Sine}45 = AD.$$

$$AD = \sqrt{OA^2 + OD^2}$$

$$= \sqrt{3438 \times 3438 + 3438 \times 3438}$$

$$= 2431$$

$$\text{So, } R\text{Sine}45 = 2431 \text{ (approx..)}$$

So, we have $R\text{Sine}15 = 890$; $R\text{Sine}30 = 1719$; $R\text{Sine}45 = 2431$; $R\text{Sine}60 = 2978$; $R\text{Sine}75 = 3321$;

6. Now, Chord 180 = Diameter of the circle. Half of this is $R\text{Sin}90$. So, $R\text{Sine}90 = 3438$.

7. Extend line ON to make Chord AR with angle ROA, 15deg. Refer image.

$$\text{Chord length } 15\text{deg} = AR$$

$$\text{So, } R\text{Sine}7.5 = AR/2$$

$$AR = \sqrt{AN^2 + NR^2}$$

$$NR = OR - ON = R - R\text{sin}75 \text{ (note the way of representing)}$$

$$= 3438 - 3321$$

$$= 117.$$

$$AN = R\text{Sine}15 = 890 \text{ (refer previous post)}$$

$$AR = \sqrt{NR^2 + (R\text{Sine}15)^2}$$

$$AR = \sqrt{(R - R\text{Sin}75)^2 + (R\text{Sine}15)^2}$$

$$2 \times R\text{Sine}7.5 = \sqrt{(R - R\text{Sine}75)^2 + (R\text{Sine}15)^2}$$

$$\text{So formula is, } 2 \times R\text{Sine}(A) = \sqrt{((R - R\text{Sine}(90 - 2A))^2 + (R\text{Sine}(2A))^2)}$$

$$\text{For, } 2 \times R\text{Sine}7.5 = \sqrt{(3438 - 3321)^2 + 890^2}$$

$$= 898$$

$$\text{So, } R\text{Sine}7.5 = 449 \text{ (approx..)}$$

8. Mark the mid-point of AR as S and join OS. Now in right angle triangle OSA.

$$R\text{Sine}(\text{angle}(SAO)) = R\text{Sine}82.5 = OS$$

$$OS = \sqrt{OA^2 - AS^2}$$

$$= \sqrt{R^2 - (R\text{Sine}7.5)^2}, \text{ refer AR above.}$$

$$\text{So formula is, } R\text{Sine}(A) = \sqrt{R^2 - (R\text{Sine}(90 - A))^2}$$

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$$\begin{aligned} \text{RSine}82.5 &= \text{Sqrt}(3438^2 - 449^2) \\ &= 3409 \text{ (approx.)} \end{aligned}$$

These above two formulae can be used to find any level of RSine values.

You it to find other RSine values shared by Aryabhatiya. It can be noted that Aryabhatiya shared approximate values of RSine, however, he has shared method of calculating square root in fraction, which can be used to calculate exact values.

Stanza 12:

प्रथमात्वापज्या अर्धात्यैरूनं खण्डितं द्वितीय अर्धम् ।
तद् प्रथमज्या अर्धाशैस्तैस्तैसूनानि शेषाणि ॥ २.१२ ॥

“The first Rsine divided by itself and then diminished by the quotient gives the second Rsine-difference. The same first Rssine diminished by the quotients obtained by dividing each of the preceding Rsines by the first Rsine gives the remaining Rsine-differences.”

This is derivation of RSine-differences. As we have seen in Gitika Pada that Aryabhata has shared RSine-difference from which we found RSine values. Now in this stanza, we will see the concept of RSine-differences.

As we have seen formulae in earlier stanza that,

$$2 \times \text{RSine}(A) = \text{Sqrt}(((R - \text{RSine}(90 - 2A))^2 + (\text{RSine}(2A))^2)$$

$$\text{RSine}(A) = \text{Sqrt}(R^2 - (\text{RSine}(90 - A))^2)$$

We will take all 24 RSines as R1, R2, R3...R24 & their differences as D1(R1 itself), D2, D3...D24. Then the formula is,

$$D(n+1) = Dn - (Rn/R1) \times (D1 - D2)$$

To arrive at approximate values, Aryabhata considered D1-D2 as 1 (225-224).

3. Interesting thoughts

Science operates on assumptions that natural causes explain natural phenomena, that evidence from natural world can inform us about those causes, and that these causes are consistent. In physics, an assumption is a statement that is accepted as truth without logical reasoning. It will not attract questions about rational behind it. However, all further reasoning should be based on it. Understanding the assumptions are very important as any of your further claims should be clear thought of considering the assumptions made and without violating them. Any claim is valid with those assumptions. Any change in assumption may change the claim. Example: $E = m \cdot (c^2)$. Following are the assumptions made and equation is valid only with these assumptions.

1. Galilean and Newtonian mechanics.
2. Galilean relativity and frame of reference.
3. Conservation of energy.

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4. Conservation of momentum.

5. Speed of light is universal

Question these assumptions to understand the correct meaning and purpose of claims made.

Why didnt we question the assumption made by Einstein that speed of light is universal?

Light travels with photons combined together in neutral form. If a neutral photon can travel at speed Of light, why cant a negative or positive charge particle accelerated in a medium covered with it's own charge polarity accelerated by opposite medium travel faster than photons?

If we throw a ball upwards with a initial velocity less than escape velocity, the ball reaches to Height, velocity becomes zero and turn back downwards to reach surface. It will touch the ground with velocity faster or slower than initial velocity depending on the height of its return travel. In a black holes, light leaves and pulled inside. So why cant photons in light Follow the same change? Why speed of light is universal?

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15. Do you know?



16. Do you know?

Madhava's Contribution:

Madhava derived the following series for the computation of π :

Madhava series for π

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

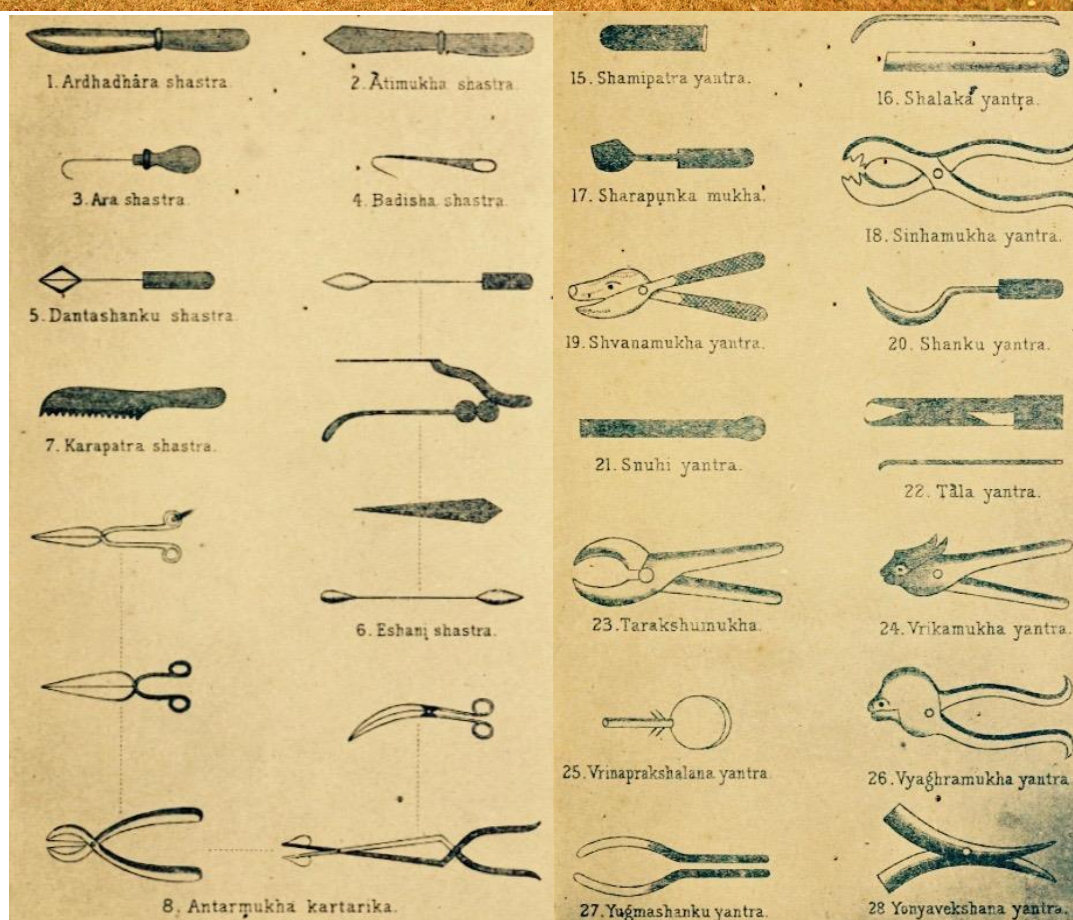
- The series is known as the Gregory series.
- Its discovery has been attributed to Gottfried Wilhelm Leibniz (1646 - 1716) and James Gregory (1638 - 1675).
- The series was known in Kerala more than two centuries before its European discoverers were born.

Using this series and several correction terms Mādhava computed the following value for π :

$$\pi = 3.1415926535922.$$

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17. Do you know?



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4. Interesting thoughts

Simple and interesting question. Can the term Birth be defined in scientific terms? Let us say, total mass of Universe is constant and energy in universe is also same. Birth must be the point at which the Universe starts expanding. Expansion term can raise a question about into Where it expands. You need space to expand. How much it can expand to comes the limits of space? Let us keep the term as growth instead of expansion. So, we can say that the point of time when universe starts growing is its birth and our measurable time cycle starts there. What happens with us? What is our birth? Mass of universe is constant. Our body weight must have existed in some form in Universe. When did we start growing from the smallest form possible? As per science, It is not that the smallest form suddenly started in Earth. It can be ever existing but started growing when the situation in Earth started supporting its growth.

Again to the universe, something is out of reach to science that it is so big, so small at the same time and out of our measurable time limits.

Let us say, the shape and size Of the universe takes a particular exact form during the birth or end of life cycle. If same situation exists during each birth, then growth of the universe has to be the same every time as there are no known external factors. So, the life we live in our current body now should have been there in each lifecycle of universe. Are we living the same life in this body which we did in our earlier life of universe?

If Universe doesn't take the same form and size during every birth then there should be an external variable outside universe for that change?

Stanza 13:

वृत्तं भ्रमेण साध्यं च चतुर्भुजं च कर्णाभ्याम् ।
साध्या जलेन समभूरध ऊर्ध्वं लम्बकेन एव ॥ २.१३ ॥

"A circle should be constructed by means of a pair of compasses; a triangle and a quadrilateral by means of the two hypotenuses (कर्णा, karna). The level of ground should be tested by means of water; and vertically by means of a plumb"

Constructing a circle, triangle and quadrilateral are as seen in our school geometry. However, note the origin of such education. It was there even during Aryabhatiya days.

It is clear that gravity is used here to obtain verticality. Due to gravity & the guna of water is to flow to the lower side, horizontal (which is perpendicular to vertical line) can be obtained.

When there is no wind, place a jar of water upon a tripod on the ground which has been made plane by means of eye or thread, and bore a fine hole at the bottom of the jar so that water may have continuous flow. Where the water falling on the ground spreads in a circle, there the ground is in perfect level; where the water accumulates

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after departing from the circle of water, there it is low; and where the water does not reach, there it is high.

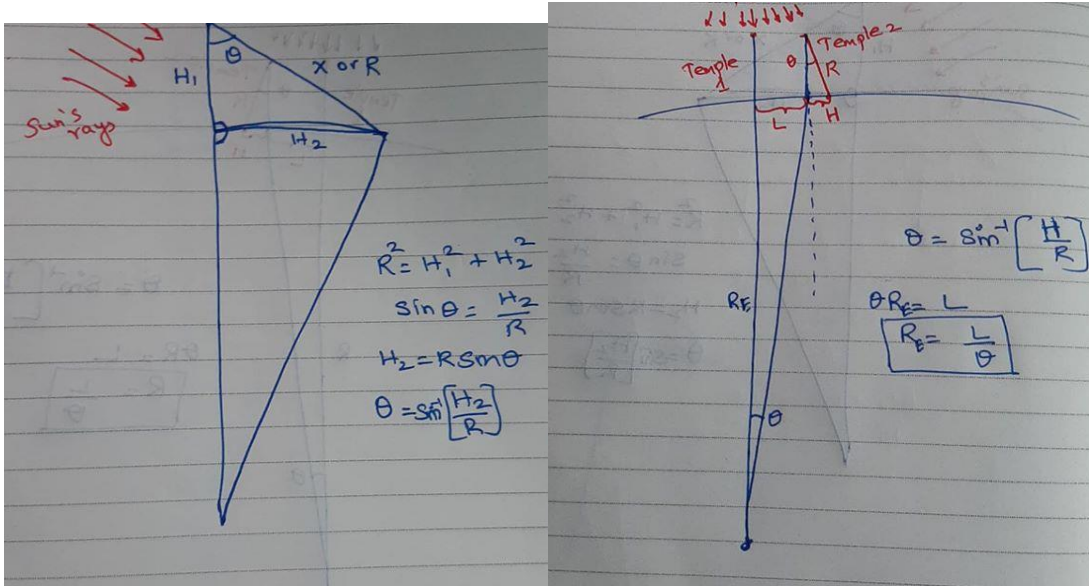
Stanza 14:

शङ्कोस्प्रमाणवर्गं छायावर्गेण संयुतं कृत्वा ।
यत्तस्य वर्गमूलं विष्कम्भ अर्धं स्ववृत्तस्य ॥ २.१४ ॥

"Add the square of the height of the gnomon to the square of its shadow. The square root of that sum is the semi diameter of the circle of drawn with the length between shadow and tip of gnomon (svavrtta, स्ववृत्तस्य)"

Gnomon means as object placed vertically, used for measuring the hours of the day using Sun's light. Now, it is obvious that vertical line (Gnomon) and the shadow will form a right-angle triangle. Here the hypotenuse formula is given here by the other two sides. It is essential to understand.

Hypotenuse² = Height of gnomon² + Height of the shadow². (refer first image)



Height here is length of the shadow. It is important to note that Aryabhata has also shared the formula for calculating the data shared by him. He has shared the Radius of Earth. This is stanza is important to understand how he calculated it. Earlier, we have seen Sine function in term of radius of circle with is Rsine. Here to calculate the Sine value from right-angled triangle, we have to consider hypotenuse as radius(R). Why it is related to shadow of any object?

Let us consider, there are two temples or tall towers, which are constructed 'L' distance apart (refer second image). Both temples have pointed tip (kumbha) & near the first temple tip on top, a person is standing. At the next temple, a person is standing below the temple. There is one more person in the scene which I will tell later. Now, when the Sun is exactly on top of the first temple (equinox. point), no shadow can be seen of the temple. Remember, temples are broad so seen no shadow is difficult as it can fall over it as well. But can be seen for the kumbha by the person standing close to it. Now, at the same time, second person marks the point of the shadow tip on the

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ground. Measure the distance (H) between the shadow tip and center of the temple & length (R) between kumbha tip and the marked point. Using R & H Sun's declination at second temple can be found.

$$\text{Sun's declination} = \text{Asin}(H/R)$$

Using Sun's declination & distance between temple (L), Radius of Earth can be found.

$$\text{Radius of Earth} = L / \text{Sun's declination.}$$

Why is a third person required? How will the second person at may around 3km away in second temple, that it is time for him to mark? Third person will ring temple bell of first temple when the person on top tell him it is time. Temple bells are loud enough to travel distances.

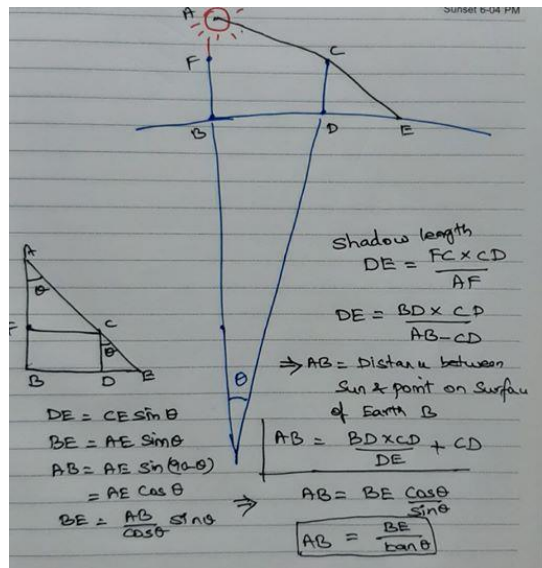
Stanza 15:

शङ्कुगुणं शङ्कुभुजाविवरं शङ्कुभुजयोर्विशेषहतम् ।
यत्लब्धं सा छाया ज्ञेया शङ्कोस्त्वमूलाह्नि ॥ २.१५ ॥

“Multiply the distance between the gnomon and the lamp post (horizontal distance between the source of light and gnomon) by the height of the gnomon and divide by the difference between the heights of the lamp post and the gnomon. The quotient should be known as the length of the shadow measured from the foot of the gnomon”

This stanza is for explaining similar triangles and its relation to shadow and light source.

In nature, light source is from Sun. In our last stanza, we saw how to calculate the radius of Earth. Now let us see the relation between radius of Earth, Shadow length and Distance between Earth and Sun.



Refer image. Let us consider a lamp post first which is installed at point B on ground and lamp at top is at point A. So, height of the light source is AB. Let us take gnomon vertically installed at point D and its tip as point C. Height of the gnomon is CD. Now,

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join BD and extend the line further till it intersects with the line made by point A & C. Let this intersecting point be E (tip of the shadow). So, we have three triangles, ABE, AFC & CDE.

All these three mentioned triangles are similar triangles. So, as the formula given by Aryabhata

Length of the shadow = Distance between the gnomon & light source x height of the gnomon / (height of the light source – height of the gnomon)

$$DE = FC \times CD / (AB - CD)$$

Now, let us consider that the light source is Sun & point A is Sun itself. Then B is the point on the surface of earth where no shadow can be seen (which lies on the line joining center of Earth & Sun). Now AB is the distance between the Sun and the point of the surface of Earth.

In the above formula, DE can be measured, CD can be measured & FC is the nothing but BD which can also be measured. We can find AB by substituting all values.

We know the angle(DCE) is Sun's declination which is equal to angle(BAE). As per last, stanza, considering R as AE in triangle ABE.

$$AB = AE \times \sin(90 - \text{Sun's declination})$$

Similarly,

$$BE = AE \times \sin(\text{Sun's declination})$$

$$\text{So, } AB = BE \times \sin(90 - \text{Sun's declination}) / \sin(\text{Sun's declination})$$

In triangle CDE, considering R as CE.

$$DE = CE \times \sin(\text{Sun's declination})$$

$$CD = CE \times \sin(90 - \text{Sun's declination})$$

$$\text{So, } \sin(90 - \text{Sun's declination}) / \sin(\text{Sun's declination}) = CD / DE.$$

We can find Sun's declination from the formula or substitute the value in the previous equation to get distance AB. Now distance between Sun & Earth is AB + Radius of Earth.

All sutras shared by our ancestors in Sanskrit looks small. But the meaning behind it when deciphered can give enormous information to us.

Stanza 16:

छायागुणितं छायाअग्रविवरं ऊनेन भाजितं कोटी ।
शङ्कुगुणा कोटी सा छायाभक्ता भुजा भवति ॥ २.१६ ॥

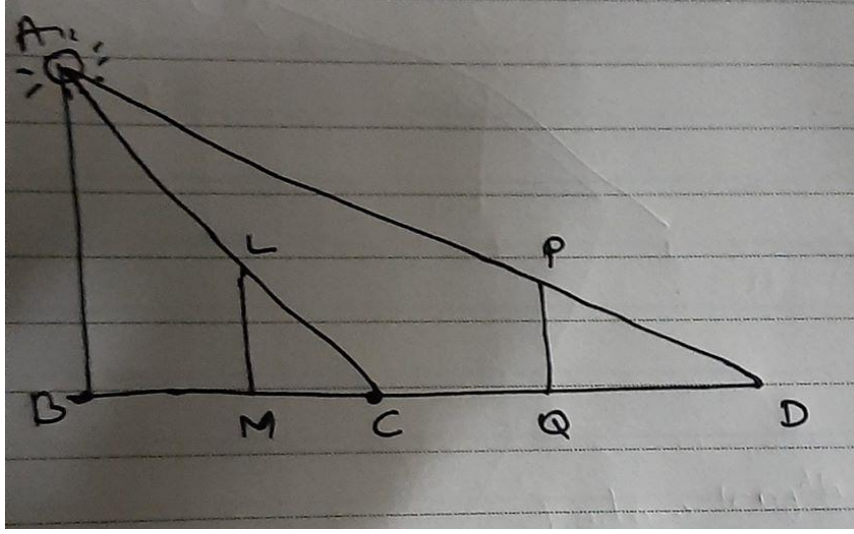
"When there are two gnomons of equal height in the same direction from the lamp post then multiply the distance between the tips of the shadows of the two gnomons by the shadow (larger or shorter) and divide by the larger shadow diminished by the shorter one. The result is the upright, ie., the distance of the tip of the (larger or shorter)

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shadow from the foot of the lamp-post. The upright multiplied by the height of the gnomon and divided by the (larger or shorter) shadow gives the height of the lamp post"

In this stanza, we don't need any data on the position of the lamppost to calculate the height of the lamp post. Note in earlier stanza, we need distance between the lamppost and the gnomon to find the height of the lamp post.

So, considering Sun as light source, we don't need to bother on the exact distance between Solar noon point and the gnomon. Let us go into the formula shared in this stanza.



Construct points as in image where A is point of light source (Sun), B is noon point on Earth, LM is gnomon 1, PQ is gnomon 2, C is shadow tip of gnomon 1 & D is shadow tip of gnomon 2.

Then, by similar triangle concept as seen in earlier stanza,

$$AB / PQ = BD / QD$$

$$AB / LM = BC / MC$$

Since gnomon length are same. $PQ = LM$.

$$BD / QD = BC / MC = CD / (QD - MC)$$

Hence,

$$AB = BD \times PQ / QD \text{ or}$$

$$AB = BC \times LM / MC$$

where AB is the distance between light source (Sun) and the local noon point.

Stanza 17:

यस्च एव भुजावर्गस्कोटीवर्गश्च कर्णवर्गस्स ।
वृत्ते शरसंवर्गस् अर्धज्यावर्गस्सखलु धनुषोस् ॥ २.१७ ॥

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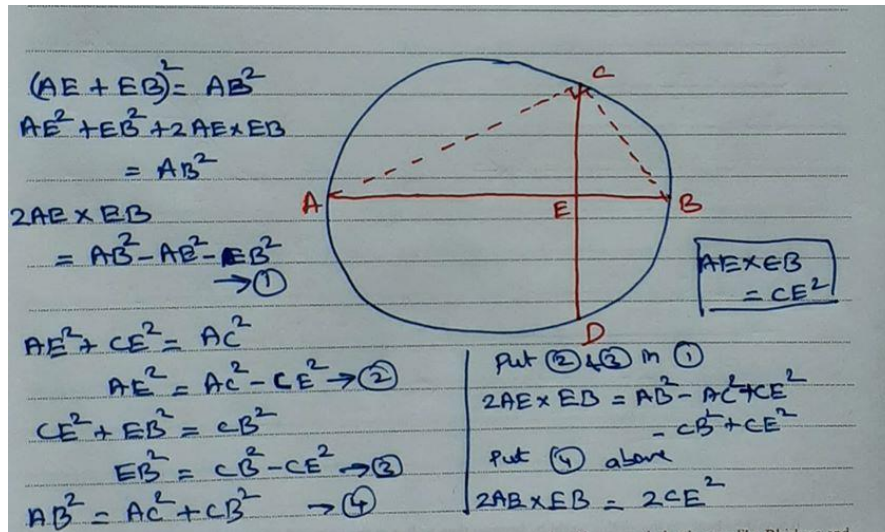
"In a right-angled triangle the square of the base plus the square of the upright is the square of the hypotenuse. In a circle, when a chord divides it into two arcs, the product of the arrows of the two arcs is certainly equal to the square of half of the chord"

First line is on the theorem of the square of the hypotenuse.

This theorem has been in India even before Aryabhata. Baudhayana of 800 BC, the author of the Baudhayama-sulba-sutra has enunciated it as

"The diagonal of a rectangle produces both (areas) which its length and breadth produce separately"

First theorem shared by Aryabhata is now universally called as Pythagoras theorem, for which, there is no real trustworthy evidence that it is actually discovered by him. It was certainly the Indian Sanata dharmis who enunciated the property of the right-angled triangle in its most general form. It is also to be noted that no other ancient nation is known to have made any attempt in this direction or have shared any such theorem which forms the basis of many trigonometry theorems. I would like to state it as,



Aryabhata's Square of hypotenuse theorem:

$$\text{Hypotenuse}^2 = \text{Length}^2 + \text{Height}^2$$

Second line of the stanza states another theorem which we would have studied in Geometry, that is, in a circle (refer image), a chord CD and a diameter AB intersect each other at right angles at E then

$$AE \times EB = CE^2.$$

This also implies that for angle length of Chord CD, Angle (ACB) is 90deg. Also, it implies that CAE & CEB are similar triangles which are oriented at 90deg.

Stanza 18:

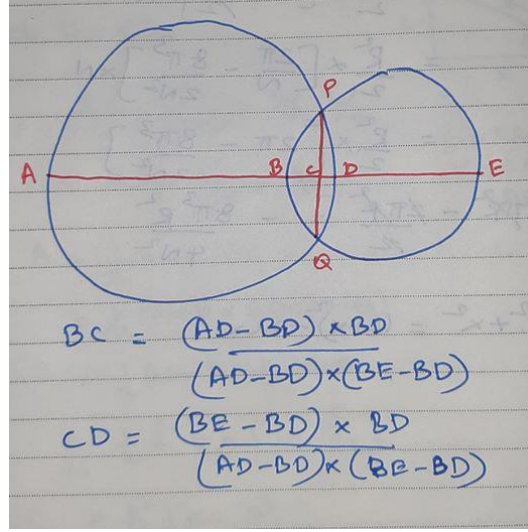
ग्रासोने द्वे वृत्ते ग्रासगुणे भाजयेत्पृथक्त्वेन ।
ग्रासऊनयोगलब्धौ संपातशरौ परस्परतस् ॥ २.१८ ॥

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“When one circle intersects another circle, multiply the diameters of the two circles each diminished by the erosion, by the erosion and divide each result by the sum of the diameters of the two circles after each has been diminished by the erosion, then are obtained the arrows of the arcs of the two circles intercepted in each other”

This stanza is about arrows of intercepted arcs of intersecting circles. This theorem is very much essential while we discuss on Solar & lunar eclipse formed by the shadows of Moon & Earth.

Refer image, Let the two circles with difference diameter AD & BE intersect. Now, let the points of intersection be P & Q. Mark point of intersection of AE & PQ as C.



As per the theorem,

$$BC = (AD - BD) \times BD / ((AD - BD) + (BE - BD))$$

&

$$CD = (BE - BD) \times BD / ((AD - BD) + (BE - BD))$$

Stanza 19:

इष्टं वि एकं दलितं सपूर्वं उत्तरगुणं समुखं मध्यम् ।

इष्टगुणितं इष्टधनं तु अथ वा आदिअन्तं पद अर्धहतम् ॥ २.१९ ॥

“Diminish the given number of terms by one, then divide by two then increase by the number of the preceding terms (if any), then multiply by the common difference, and then increase by the first term of the series, then the result is the arithmetic mean (of the given number of terms). This multiplied by the given number of terms is the sum of the given terms. Alternatively, multiply the sum of the first and last terms of the series or partial series which is to be summed up by half the number of terms”

Interesting way to share formula for arithmetic series which can give so many formulae. Let us see what is Arithmetic series before we go inside the information shared in above stanza.

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Arithmetic series is a series in which difference between consecutive terms will be a constant. A simple well know arithmetic series is given below which is the first thing we learn in Mathematics.

1,2,3,4,5,6.....

In above series, difference between consecutive term is constant i.e.,

$$(2-1) = (3-2) = (4-3) = (5-4) = (6-5) = 1$$

In general, this can be written as

a, (a+d), (a+2d), (a+3d), ... (a+(n-1)d)

Here, 'a' is the starting term and 'd' is the difference between consecutive terms.

Series 1: If a = 1 & d = 1 then 1, 2, 3, 4, 5.....(1+(n-1)x1)

Series 2: If a = 2 & d = 2 then 2, 4, 6, 8, 10(2 + (n-1) x 2) {seen multiplication table of 2?}

Series 3: If a = 3 & d = 3 then 3, 6, 9, 12, 15(3 + (n-1) x 3) {multiplication table of 3!}

So, there are many arithmetic series as seen above and it is importance to know how to identify the series and importance of terms related to this series.

Arithmetic sum is simple, sum of terms in series. We will see the importance of this formula when we see error accumulation in Kalakriya pada.

1. As per the stanza, formula for arithmetic mean of 'n' terms is

$$\text{Arithmetic mean of } \{(a+pd), (a+(p+1)d), \dots, (a+(p+(n-1))d)\} = a + ((n-1)/2 + p) \times d$$

2. As per the sanza, formula for arithmetic sum of 'n' terms

Arithmetic mean = n x arithmetic mean. i.e,

$$(a+pd) + (a+(p+1)d) + \dots + (a+(p+(n-1))d) = n \times (a + ((n-1)/2 + p) \times d)$$

3. Also, alternatively, if we know the first and last term of the series then,

$$\text{Sum of the terms or Arithmetic sum} = (n/2) \times (a + L)$$

Where, a is the first term and L is the last term.

There are so many Indian scripts on Arithmetic series, its sum & mean which was shared way back. Such series were never shared earlier in any of the work other than Indian origin.

In the Brhaddevata (500-400BC), we have result of

$$2 + 3 + 4 + \dots + 1000 = 500499.$$

Here, a = 2, p = 0, d = 1 & L = 1000,

From above formula, we know that

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$$L = a + (p+(n-1) \times d)$$

$$1000 = 2 + (0 + (n-1) \times 1)$$

$$n = 999.$$

Now, we will check with different formula shared to check the answer.

$$\begin{aligned} 1. \text{ Sum} &= n \times (a + ((n-1)/2 + p) \times d) \\ &= 999 \times (2 + ((999-1)/2 + 0) \times 1) \\ &= 999 \times (2 + 499) \\ &= 500499. \end{aligned}$$

$$\begin{aligned} 2. \text{ Sum} &= (n/2) \times (a + L) \\ &= (999/2) \times (2 + 1000) \\ &= 1000998/2 \\ &= 500499. \end{aligned}$$

In our school, we would have studied that arithmetic sum formula for n terms starting from 1 is $n \times (n+1)/2$. This formula is from the above formula shared by Aryabhata.

Stanza 20:

गच्छसष्टोत्तरगुणिताद्धि-गुणाद्युत्तरविशेषवर्गयुतात् ।
मूलम् द्विगुणाद्यूनं स्वोत्तरभजितं सरूपार्धम् ॥ २.२० ॥

“The number of terms are obtained as follows: Multiply the sum of the series by eight and by the common difference, increase that by the square of the difference between twice the first term and the common difference, and then take the square root; then subtract twice the first term, then divide by the common difference, then add one to the quotient, and then divide by two”

This stanza is about number of terms in an arithmetic progression.

Let us consider the series as

a, (a+d), (a+2d), (a+3d), (a+4d),n terms

So, sum of the series is,

$$S = a + (a+d) + (a+2d) + (a+3d) + (a+4d) +n \text{ terms}$$

Then,

$$n = (1/2) \times [(\sqrt{8dS + (2a-d)^2} - 2a)/d + 1]$$

Let us take an example, (same example as discussed in earlier stanza)

$$2 + 3 + 4 + + 1000 = 500499.$$

Here, a = 2; d = 1; S = 500499. Let us find ‘n’ with the formula given.

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$$\begin{aligned}n &= (1/2) \times [(\sqrt{8 \times 1 \times 500499 + (2 \times 2 - 1)^2} - 2 \times 2)/1 + 1] \\&= (1/2) \times [(\sqrt{4003992 + (3)^2} - 4)/1 + 1] \\&= (1/2) \times [(\sqrt{4004001} - 4)/1 + 1] \\&= (1/2) \times [(2001 - 4)/1 + 1] \\&= (1/2) \times [(1997) + 1] \\&= (1/2) \times [1998] \\&= 999\end{aligned}$$

Now, see the series, last term is 1000 and there is no 1, so number of terms is 999. Surprisingly,

In this formula, the term inside the sqrt is a perfect square, $8dS + (2a-d)^2$.

In case, of $a = 2$; $d = 1$; $S = 500499$, term inside square root is 4004001. Square root of which is 2001.

If in case, of $a = 2$; $d = 1$; $S = 501500$ (added 1001), term inside square root will be 4012009. Square root of which is 2003 (This square root follows an arithmetic progression with 'd' as 2).

5. Interesting thoughts

Interestingly, Happiness is competitive. If you want to be Happy, you need to experience/enjoy sadness. We enjoy rain because we have summer. Earth's tilt (can be an imperfection) which gives us this enjoyment. No season if no tilt in Earth's axis.

But, there are something/feeling which are not time bound like the feeling you experience by devoted kankaryam to God. These remain beyond reach for science. Similarly, numbers zero and infinity are not just numbers. Practical example cannot be given for them without considering Time or other dimension. Can you explain the life of Energy in Universe in terms of time? Rule in science says Energy can neither be created nor by destroyed. Whether energy not part of universe if universe has a birth time?

Stanza 21:

एकोत्तराद्युपचितेसगच्चादि-एकोत्तर-त्रि-संवर्गस् ।
षष्ठभक्तस्सच्चितिघनस्स एकपदघनस्विमूलस्वा ॥ २.२१ ॥

"Of the series (upaciti) which has one for the first term and one for the common difference, take three terms in continuation, of which the first is equal to the given number of terms, and find their continued product. That product, or the number of terms plus one subtracted from the cube of that, divided by 6, gives the citighana"

Let us understand the terms mentioned in the stanza. Citi means series or pile. ekottaradi-upaciti is series starting with 1 and common difference as 1. Sum of series

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is Sankalita. Series made by the sum of Sankalita is sankalita-sankalita. Citighana is solid balls of a pile in the shape of a pyramid on a triangular base.

Now, let us see the above terms in numbers

ekottaradi-upaciti: $1 + 2 + 3 + \dots + n$

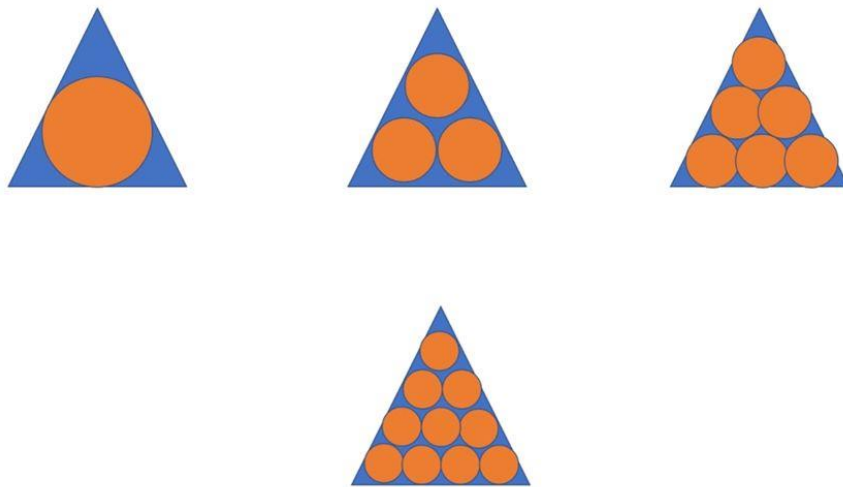
sankalita-sankalita: $1 + (1+2) + (1+2+3) + \dots$

This stanza gives the sum of sankalita-sankalita series as

Sum of sankalita-sankalita series = $n \times (n+1) \times (n+2) / 6$ or

Sum of sankalita-sankalita series = $[(n+1)^2 - (n+1)] / 6$

Let us take a practical example as shared by Bhaskara where this can be applied.



There are three pyramid piles having respectively 5, 8 and 14 layers which are triangular. Tell me the number of units (balls) in each of them.

The above citighana is a series of figurate numbers. The Hindus are known to have obtained the formula from the sum of series of natural numbers as early as the fifth century BC. It cannot be said with any certainty whether the Hindus in those times used the representation of the sum by triangles or not. The subject of piles of shots and other things has been given great importance in the Hindu works, of which all contain a section dealing with citi (piles). It will not be a matter of surprise if the geometrical representation of figurate numbers is traced to Hindu sources.

Formula shared in the stanza is

Sum of sankalita-sankalita series = $n \times (n+1) \times (n+2) / 6$ or

Sum of sankalita-sankalita series = $[(n+1)^2 - (n+1)] / 6$

Consider the example of balls placed inside triangular pyramid. Refer image.

Number of layers of stack is 4. So as per the formula, total number of balls are

1. Total number of balls = $4 \times (4+1) \times (4+2) / 6$

$$= 4 \times 5 \times 6 / 6$$

$$= 20$$

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$$2. \text{ Total number of balls} = [(4+1)^2 - (4+1)] / 6$$

$$= [25 - 5] / 6$$

$$= 20/6 \text{ (Not the answer)}$$

So, we need to consider first formula answer in this case, which is 20. Now count the number of balls shown in top view of the picture. First layer, 1; second layer, 3; third layer, 6 and fourth layer, 10. So, number of balls are $(1 + 3 + 6 + 10) = 20$.

Stanza 22:

स एकसगच्छपदानां क्रमात् त्रिसंवर्गितस्य षष्ठसंशस् ।

वर्गचितिघनस्सस् भवेत्चितिवर्गस्थनचितिघनस्च ॥ २.२२ ॥

“The continued product of the three quantities, viz., the number of terms plus one, the same increased by the number of terms, and the number of terms when divided by 6 gives the sum of the series of squares of natural numbers (वर्गचितिघन, vargacitighana). The square of the sum of the series of natural numbers (चिति, citi) gives the sum of the series of cubes of natural numbers (वर्गस्थनचितिघन, ghanacitighana)”

The term vargacitighana is used in the sense of the sum of the series shown below

$$1^2 + 2^2 + 3^2 + \dots + n^2 = [n \times (n+1) \times (2n+1)]/6$$

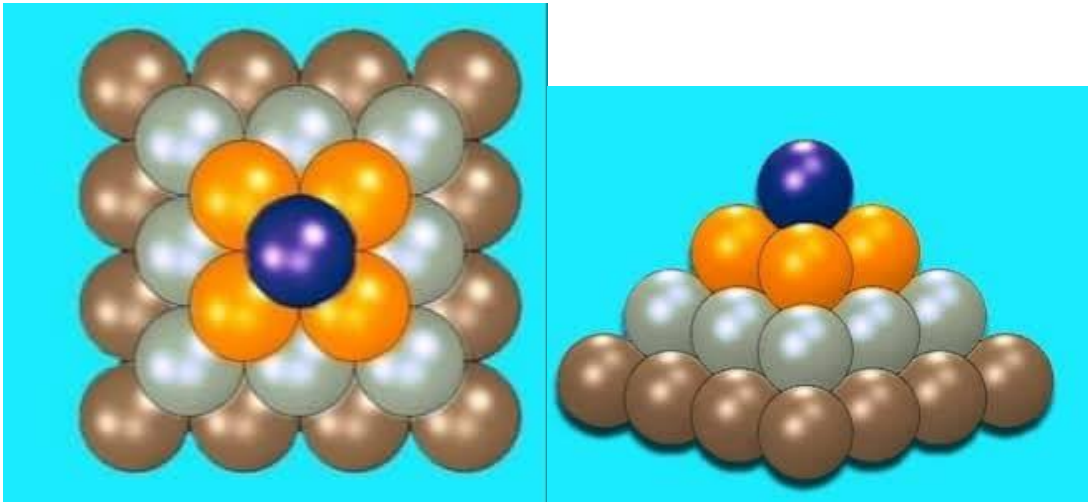
Vargacitighana literally means “the solid contents of a pile (of balls) in the shape of a pyramid on a square base. It is so constructed that there is 1 ball in the topmost layer, 2^2 balls in the next lower layer, 3^2 balls in the further next lower layers and so on.

Refer first image, $n = 4$ as there are four layers. Count the number of balls in the pyramid which can also be calculated as per below:

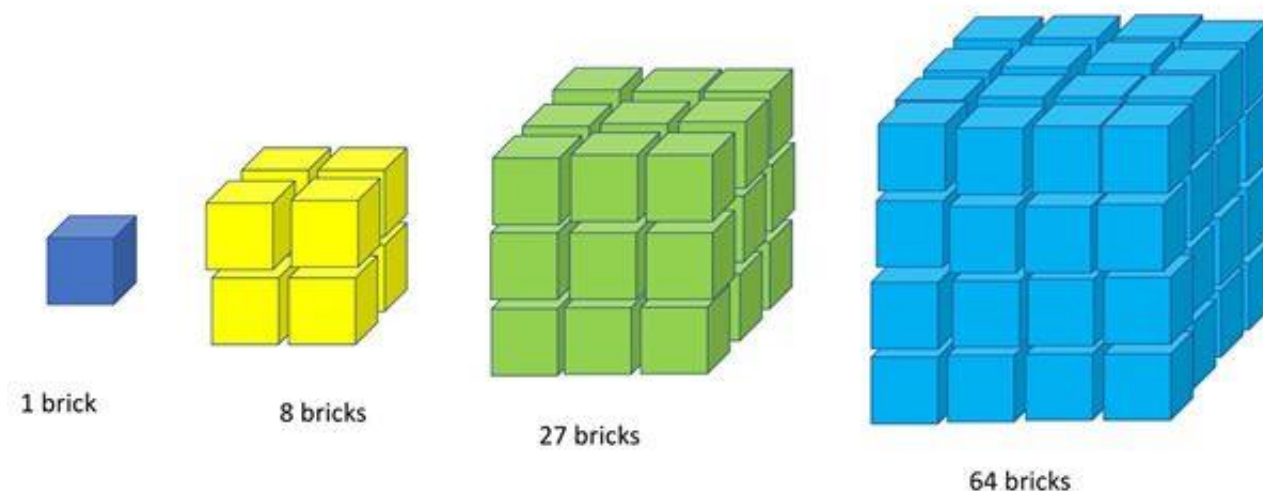
$$\text{Number of balls in four layer pyramid} = 4 \times (4+1) \times (2 \times 4 + 1) / 6$$

$$= 4 \times 5 \times 9 / 6$$

$$= 30.$$



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Next formula is for ghanacitighana which means “the solid contents of a pile of cuboidal bricks in the shape of a pyramid having cuboidal layers”. It is so constructed that there is 1 brick in the topmost layer, 2^3 brick in the next lower layer, 3^3 bricks in the further next lower layer, and so on.

$$1^3 + 2^3 + 3^3 + \dots + n^3 = [n \times (n+1)/2]^2$$

Refer second image. $n = 4$ as there are four layers. Count the number of balls in the pyramid which can also be calculated as per below (note that there are same number of cubes below the next bottom layer which are hidden by the layer above it)

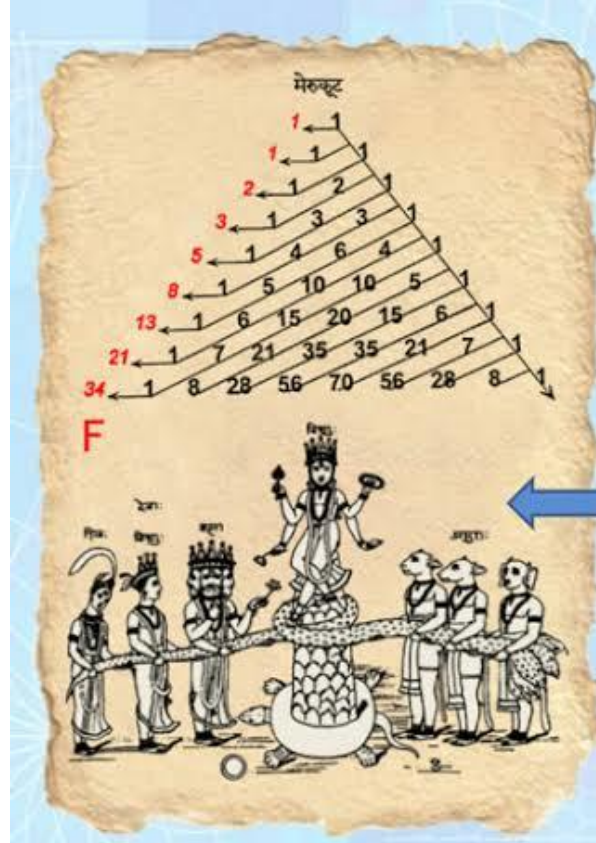
$$\text{Number of cuboids} = [4 \times 5 / 2]^2$$

$$= 10^2$$

$$= 100.$$

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18. Do you know?



The treatise **Indian mathematics Pingala** contains the most ancient mention of Fibonacci-numbers, binary system, binomial coefficients and the Pascal's triangle (or sacred pyramid **Mahatrat-Meru**).

Sanskritist **Michael Mikhajlov** has proved, that Vedic texts can be read with the help of **mathematical and astronomical Pingala's code**.

19. Do you know?

Baudhaya's formula for square root of 2:

समस्य द्विकरणौ। प्रमाणं तृतीयेन वर्धयेत्
तच्चतुर्थेनात्म चतुस्त्रिंशोनेन सविशेषः ॥

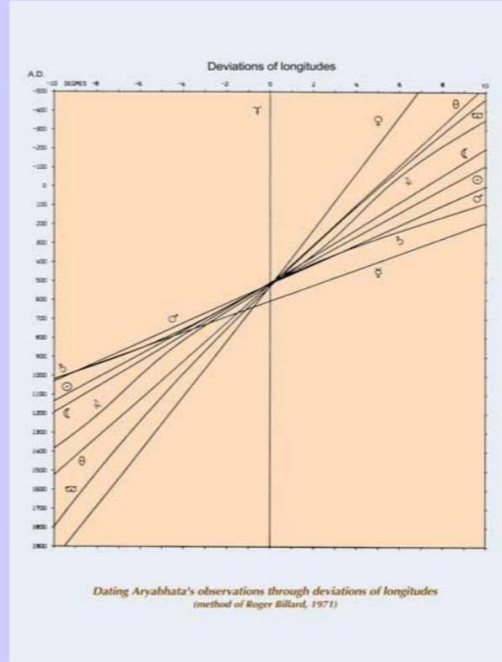
- (Bau. i, 61-2, Ap. i, 6)

$$\text{i.e. } \sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34}$$

20. Do you know?

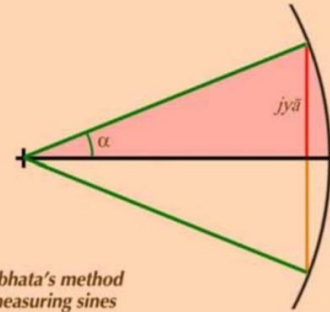
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In 1971 Roger Billard (a French mathematician and Sanskritist) did a statistical study of the deviations of longitudes of Āryabhata's observations of planets. He proved that the deviations were smallest around 500 CE, which is the date of the *Āryabhatīya*. This gave the lie to scholars who claimed that Āryabhata had borrowed his table of planetary positions from Babylonian astronomers.



21. Do you know?

makhi bhakhi phakhi dhakhi ṇakhi ñakhi
ṇakhi haṣṭha skaki kiṣga ghakhi kighva |
ghlaki kigra hakya dhaki kica
sga śjha ṇva kla pta pha cha kala-ardha-jyāḥ ||
(Āryabhatīya, verse 12)



Aryabhata's method
for measuring sines

Āryabhata also ...

- Gave a table of sines (above): 24 values for the first quadrant in increments of 3.75° (all values correct to 3 or 4 significant figures).
- Proposed that $\pi = 62832 / 20000 = 3.1416$, adding that it was an "approximate" value.
- Gave for the first time the formula for the area of a triangle.
- Solved in integers linear indeterminate equations of the type $ax + c = by$ through the *kuttaka* or "pulverizing" method.

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Stanza 23:

सम्पर्कस्य हि वर्गात् विशोधयेतेव वर्गसम्पर्कम् ।
यत्तस्य भवति अर्धं विद्यात्गुणकारसंवर्गम् ॥ २.२३ ॥

“From the square of the sum of the two factors subtract the sum of their squares. One half of that difference should be known as the product of the two factors”

Formula shared in the stanza is,

$$A \times B = [(A+B)^2 - (A^2+B^2)]/2$$

Looks familiar? let us rearrange the formula,

$$A \times B = [(A+B)^2 - (A^2+B^2)]/2$$

$$2 \times A \times B = [(A+B)^2 - (A^2+B^2)]$$

$$A^2 + B^2 + 2AB = [(A+B)^2]$$

$$(A+B)^2 = A^2 + B^2 + 2AB.$$

Now, looks familiar? this is $(a+b)^2$ formula which we learnt in our school. All these are from Aryabhatiya.

Stanza 24:

द्विकृतिगुणात्संवर्गात् द्विअन्तरवर्गेण संयुतात्मूलम् ।
अन्तरयुक्तं हीनं तद्गुणकार द्वयं दलितम् ॥ २.२४ ॥

“Multiply the product by four, then add the square of the difference of the two quantities, and then take the square root. Set down this square root in two places. In one place, increase it by the difference of the two quantities, and in the other place decrease it by the same. The results thus obtained, when divided by two, give the two factors of the given product”

This stanza is on finding factors if their difference and product is known. Actually, this is combination of two quadratic equations. Let us see the formula shared in the stanza.

If,

$$\text{Eqn1: } x - y = a$$

$$\text{Eqn2: } xy = b$$

then as per the formula given by Aryabhata,

$$x = [\sqrt{4b + a^2} + a]/2$$

$$y = [\sqrt{4b + a^2} - a]/2$$

Now, let

Let us see what it is in modern terms. As the solution for x doesn't have y and y doesn't have x, Equation 1 & 2 can be combined and represented with x or y alone. We will take one such form in terms of x.

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From eqn 2: $y = b/x$

Substitute this in eqn 1: $x - b/x = a$

$$(x^2 - b) / x = a$$

$$x^2 - b = ax$$

$$x^2 - ax - b = 0$$

Solution of a quadratic equation in modern terms is,

$$\text{Quadratic equation: } AX^2 + BX + C = 0$$

$$X = [-B \pm \sqrt{B^2 - 4AC}]/(2A)$$

If A is 1, B is -a & C is -b then

$$X = [a \pm \sqrt{a^2 + 4b}]/(2)$$

This is the same formula shared by Aryabhata explaining the purpose of + & - in the solution of a quadratic equation with two equation & two factors. Also, it is easy to implement this in problems which is more practical approach. Which is to compare with a practical problem? Whether difference of factor and their product or a quadratic equation?

Stanza 25:

मूलफलं सफलं कालमूलगुणं अर्धमूलकृतियुक्तम् ।
तद्मूलं मूल अर्धऊनं कालहतं स्वमूलफलम् ॥ २.२५ ॥

“Multiply the interest on the principal plus the interest on that interest by the time and by the principal; then add the square of half the principal; then take the square root; then subtract half the principal; and then divide by the time: the result is the interest on the principal”

The problem envisaged is: A principal P is lent out at a certain rate of interest per month. At the expiry of one month, the interest 'I' which accrues on P in one month is given on loan at the same rate of interest for T months. After T months 'I' amounts to A. The problem is to find 'I' when A is given.

Here, A is accumulated interest in the same monthly interest rate and 'I' is the interest of a month.

Solution to this problem given by Aryabhata is given below:

$$I = [\sqrt{PTA + (P/2)^2} - (P/2)]/T.$$

It can be seen that even during Aryabhata days, money lenders were deducting interest at the outset while lending money. They were also aware of rudiments of compound interest. Also, it can be seen that the formula is for the interest as even when we want to get loan we check only the interest rate which is of our interest. This is a form of quadratic equation hence Aryabhata has shared it after sharing formula on quadratic equation which we saw in earlier stanza.

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Stanza 26:

त्रैराशिकफलराशिं तं अथ इच्छाराशिना हतं कृत्वा ।
लब्धं प्रमाणभजितं तस्मादिच्छाफलं इदं स्यात् ॥ २.२६ ॥

“In the rule of three, multiply the fruit (फल, phala) by the requisition (इच्छा, iccha) and divide the resulting product by the argument (प्रमाण, pramana). Then is obtained the fruit corresponding to the requisition(icchaphala)”

This is an interesting formula which can be used under different areas. Let us see some examples.

1. If A number of pens cost P rupees, then How much B number of pens will cost?

Here A is the argument, P is the fruit and B the requisition, So, required answer is $[(B \times P) / A]$ rupees.

2. If the interest on Rs. 100 for 2 months is R. 5 then the interest on Rs. 25 invested for 8 months. Here, we have two arguments, Rs. 100 & 2 months; and two requisition, Rs. 25 & 8 months. The fruit is Rs. 5. So the required answer is,

$(25 \times 8 \times 5) / (100 \times 2)$ which is same 5 rupees.

Stanza 27:

छेदास्परस्परहतास् भवन्ति गुणकारभागहाराणाम् ।

छेदगुणं सछेदं परस्परं तत्सवर्णत्वम् ॥ २.२७ ॥

“The numerators and denominators of the multipliers and divisors should be multiplied by one another. Multiply the numerator as also the denominator of each fraction by the denominator of the other function; then the given fractions are reduced to a common denominator”

Above stanza gives two formula as given below. These are simple formula which we had studied in our school.

1. From first line (Simplification of quotients of fractions): $[a/b]/[c/d] = ad/bc$

2. From second line (Reduction of two fractions to a common denominator):
 $(a/b) + (c/d) = (ad + bc) / bd$

Stanza 28:

गुणकारास्भागहरास्भागहरास्ते भवन्ति गुणकारास् ।
यस्क्षेपस्सपचयसपचयस्क्षेपस्च विपरीते ॥ २.२८ ॥

“In the method of inversion multipliers become divisors and divisors become multipliers, additive becomes subtractive and subtractive becomes additive”
Simple formula on method of inversion to find the unknown in the equation.

Example: A number is multiplied by 2; then increased by 1; then divided by 5; then multiplied by 3; then diminished by 2; and then divided by 7; the result is 1. Find what is the initial number.

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To find the initial number, start from reverse order and change operation as per stanza

1 multiplied by 7; add 2; divide by 3; multiply by 5; subtract 1; divide by 2 then we get the number which is 7. So the initial number is 7.

Question:

$$((((((Y \times 2) + 1) / 5) \times 3) - 2) / 7) = 1$$

To find Y

$$Y = ((((((1 \times 7) + 2) / 3) \times 5) - 1) / 2) = 7.$$

Stanza 29:

राशिकुनं राशिकुनं गच्छधनं पिण्डितं पृथक्त्वेन ।

वि एकेन पदेन हतं सर्वधनं तत् भवति एवम् ॥ २.२९ ॥

"The sums of all combinations of the unknown quantities except one which are given separately should be added together; and the sum should be written down separately and divided by the number of unknown quantities less one: the quotient thus obtained is certainly the total of all the unknown quantities. This total severally diminished by the given sums gives the various unknown quantities"

We are back to series again. This stanza is on finding unknown quantities from sums of all but one.

If,

$$(x_1 + x_2 + \dots + x_n) - x_1 = a_1$$

$$(x_1 + x_2 + \dots + x_n) - x_2 = a_2$$

.

$$(x_1 + x_2 + \dots + x_n) - x_n = a_n$$

Then,

$$x_1 + x_2 + \dots + x_n = (a_1 + a_2 + \dots + a_n) / (n-1)$$

So that,

$$x_1 = (a_1 + a_2 + \dots + a_n) / (n-1) - a_1$$

$$x_n = (a_1 + a_2 + \dots + a_n) / (n-1) - a_n$$

$x_1, x_2, \dots, x_n$ being the unknown quantities and $a_1, a_2, \dots, a_n$ are the given sum without considering the unknown quantity.

Stanza 30:

गुलिकाअन्तरेण विभजेत् द्वयोस्पुरुषयोस्तु रूपकविशेषम् ।

लब्धं गुलिकामूल्यं यदि अर्थकृतं भवति तुल्यम् ॥ २.३० ॥

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“Divide the difference between the rupakas with the two persons by the difference between their gulikas. The quotient is the value of one gulika, if the possessions of the two persons are of equal value”

Two persons are equally rich. Of them, one possesses a gulikas and b rupakas (coins), and the other possesses ‘c’ gulikas and ‘d’ rupakas. The rule shared here is to find the value of one gulika in terms of rupakas.

That is,

$$ax + b = cx + d$$

then,

$$x = (d-b)/(a-c).$$

Stanza 31:

भक्ते विलोमविवरे गतियोगेन अनुलोमविवरे द्वौ ।
गतिअन्तरेण लब्धौ द्वियोगकालौ अतीताइष्यौ ॥ २.३१ ॥

“Divide the distance between the two bodies moving in the opposite directions by the sum of their speeds, and the distance between the two bodies moving in the same direction by the difference of their speeds; the two quotient will give the time elapsed since the two bodies met or to elapse before they will meet”

This stanza is on Meeting of two moving bodies. Relativity between two moving bodies.

1) When the two bodies are moving in opposite direction: If the bodies are facing each other i.e., if they have not already met, the distance between them when divides by the sum of their velocities will give the time of elapse before they meet.

If the bodies have already met and moving away from each other, the distance between then when divided by the sum of their velocities will give the time elapsed since they met each other.

Time to meet = (Distance between two moving bodies)/ (Velocity of body1 + Velocity of body 2)

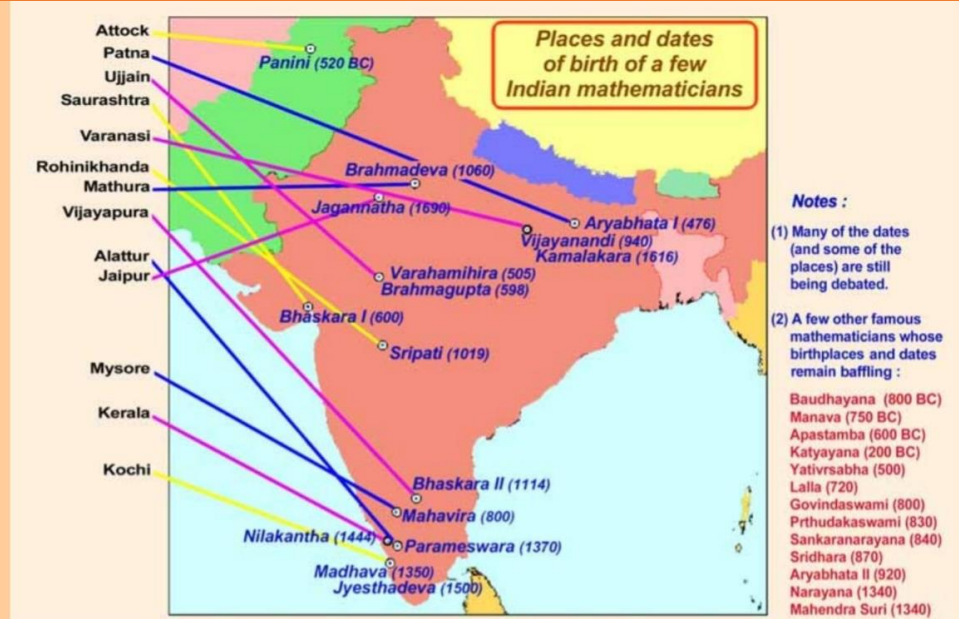
2) When two bodies are moving in the same direction: If the fast-moving body is behind, ie., if they have not already met, the distance between them when divided by the difference of their velocities will give the time to elapse before they meet.

If the slow-moving body is behind i.e, if they have already met, the distance between them when divided by the difference of their velocities will give the time elapsed since they met each other.

Time to meet = (Distance between two moving bodies)/ (Velocity of body1(fast) - Velocity of body 2(slow))

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22. Do you know?



Early Indian scientists

- This map (adapted from the website of St. Andrews University, Scotland) lists the main figures of early Indian science. (The exact place or epoch of many of them remains uncertain).
- Note the shift to the South, especially Karnataka and Kerala, after the 12th century.

23. Do you know?

Āryabhata was a brilliant scientist who lived at Kusumapura (probably today's Patna). In 499 CE, he wrote the *Āryabhaṭīya*, a brief but extremely important treatise of mathematics and astronomy, at the age of 23! A few highlights:

Āryabhata about the earth:

- The earth is a rotating sphere: the stars do not move, it is the earth that rotates.
- Its diameter is 1,050 *yojanas*. Its circumference is therefore $1050 \times 13.6 \times \pi = 44,860$ km, about 12% off. (1 *yojana* = 8,000 human heights)



Āryabhata on eclipses:

"The moon eclipses the sun, and the great shadow of the earth eclipses the moon."

(*Āryabhaṭīya*, IV.37)

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24. Do you know?

Ancient Indians conceived the infinity of time and space:

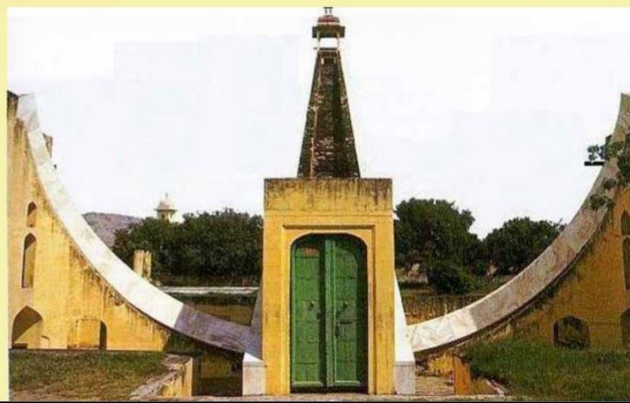
- Cyclic time.
- Limitless space — Bhâskara I: “The sky is beyond limit; it is impossible to state its measure”.
- The concept of infinity underlies much of later Indian science: Brahmagupta first spelt out the mathematical definition of infinity.
- S. Ramanujan: “The man who knew infinity” is the title of one of his biographies.

25. Do you know?



Ghati yantra bowl floating in water

- *Ghati yantra*, a type of water clock: the bowl, with a small hole at its bottom, sinks after 24 mn (a unit of time called *ghati*, equal to $1/60^{\text{th}}$ of a day).
- Ancient texts refer to various other devices (gnomons, sun dials etc.) which have disappeared, but point to a long tradition of observation.
- (Below:) A sun dial (Jantar Mantar, Jaipur, 18th century). Such massive structures are found only in recent times; ancient observatories must have consisted of simple implements made of wood.



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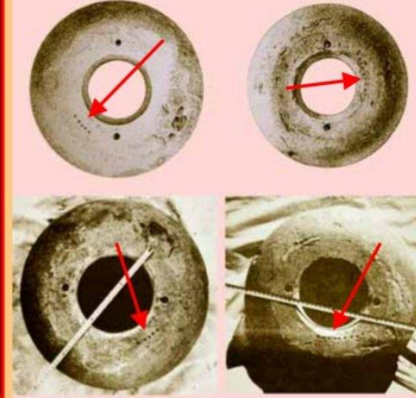
26. Do you know?

1. Indus-Sarasvati Civilization

First steps of technology and science in the protohistoric era*



The east-west alignment of the main streets of Mohenjo-daro's "citadel" (or acropolis, *left*) was based on the Pleiades star cluster (*Krittika*), which rose due east at the time; it no longer does because of the precession of the equinoxes. (German archaeologist Holger Wanzke, "Axis systems and orientation at Mohenjo-daro", 1987)



The mystery of Mohenjo-daro's "ring stones" (*above right*): the small drilled holes (see red arrows), showed the stones were used to track the path of the sun through the year, as seen from Mohenjo-daro. Such evidences demonstrate the first steps in observational astronomy. There are other hints, such as possible astronomical symbolism on a few seals. (Finnish scholar Erkkä Maulan, "The Calendar Stones from Mohenjo-daro", 1984)

* For technology, please see separate pdf file on this civilization.

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Stanza 32 & 33:

अधिकाग्रभागहारं छिन्द्यातूनाग्रभागहारेण ।
शेषपरस्परभक्तं मतिगुणं अग्रान्तरे क्षिप्तम् ॥ २.३२ ॥
अधसुपरिगुणितं अन्त्ययुज्जनाग्रछेदभाजिते शेषम् ।
अधिकाग्रछेदगुणं द्विछेदाग्रं अधिकाग्रयुतम् ॥ २.३३ ॥

“Divide the divisor corresponding to the greater remainder by the divisor corresponding to the smaller remainder. Discard the quotient. Divide the remainder obtained and the divisor by one another until the number of quotients of the mutual division is even and the final remainder is small enough. Multiply the final remainder by an optional number and to the product obtained add the difference of the remainders corresponding to the greater and smaller divisors; then divide this sum by the last divisor of the mutual division. The optional number is to be so chosen that this division is exact. Now place the quotient of the mutual division one below the other in a column; below them write the optional number and underneath it the quotient just obtained. Then reduce the chain of number which have been written down one below the other, as follows: multiply by the last but one number in the bottom the number just above it and then add the number just below it and then discard the lower number. Repeat this process until there are only two numbers in the chain. Divide the upper number by the divisor corresponding to the smaller remainder, then multiply the remainder obtained by the divisor corresponding to the greater remainder: the result is the dvicchedagram(द्विछेदाग्रं) i.e., the number answering to the two division. This is also the remainder corresponding to the divisor equal to the product of the two divisions.”

These stanzas are on problems of residual and Non-residual pulveriser. Let us take an example to understand interesting concept of solving such problems.

Problem1: Find the number which yields 5 as the remainder when divided by 8, 4 as the remainder when divided as remainder when divided by 9.

By the process shared by Aryabhata, we will get smallest positive integer which gives specified remainder when divided by given numbers.

Let 'N' be the required number. Then,

$N = 8a + 5$, here 8 is divisor and 5 is remainder.

$N = 9b + 4$, here 9 is divisor and 4 is remainder.

Among, the above two, greater remainder is 5 and divisor corresponding to the greater remainder is 8.

Difference between the remainder is $(5-4) = 1$.

Divide 5 by 4, which will give quotient 1 & remainder 1. Divide 1 by 1 which will give quotient 1 & remainder 0.

Quotients are: 1, 1. Number of quotients: 2 (only even number to be considered, if odd remove the 1st).

Let us consider 1 as the optional number.

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Now multiply last remainder 0 with optional number and add 1(difference of greater and smaller remainders), $(0 \times 1 + 1) = 1$.

Divide the result 1 by 1, the final divisor of the mutual division, the quotient is 1. As per the process, write the quotients of the mutual division as below.

1, 8, 1, 1. (Next process: multiply by the last but one number in the bottom the number just above it and then add the number just below it and then discard the lower number. Repeat this process until there are only two numbers in the chain.)

1, $(8 \times 1 + 1)$, 1, discard last 1: 1, 9, 1

Again, $(1 \times 9 + 1)$, 1: 10, 9. Last two numbers are 10 & 9.

Now, divide 10 by 9, quotient 1, remainder 1. Multiply this remainder 1 by 8, the divisor corresponding to the greater remainder, we get 8. Adding the greater remainder 5 to this 8, we get 13.

13 is the number which divided by 8 leaves 5 as remainder and divided by 9 leaves 4 as remainder. This number is called dvicchedagram (द्विच्छेदाग्रं).

This process is important in finding "aharganas" of planets.

Problem 2: Find the number which yields 19 as the remainder when divided by 45, 15 as the remainder when divided as remainder when divided by 29. So, $N = 45a + 19$, here 45 is divisor and 19 is remainder.

$N = 29b + 15$, here 29 is divisor and 15 is remainder.

Among, the above two, greater remainder is 19 and divisor corresponding to the greater remainder is 45.

Difference between the remainder is $(19 - 15) = 4$.

Now, divide 45 by 29, note down the quotient, 1 and remainder, 16.

Divide, 29 by 16, note down the quotient, 1 and remainder, 13.

Divide, 16 by 13, note down the quotient, 1 and remainder, 3.

Divide, 13 by 3, note down the quotient, 4 and remainder, 1.

Divide, 3 by 1, note down the quotient, 3 and remainder, 0.

Quotients: 1, 1, 1, 4, 3

Number of quotients: 4 (even required, so remove first)

Let us consider optional number as 2.

Last quotient is 3. Multiple 3 with optional number as $3 \times 2 = 6$. Add difference in remainder 4 as $6 + 4 = 10$. Deduce as given below.

1,1,4,3,2,4

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1,1,4,10,2

1,1,42,10

1,52,42

94,52

Last two numbers are 94 & 52. Divide 94 by 29 (divisor of smaller remainder), we get quotient 3 & remainder 7. Multiple 7 with divisor of the greatest remainder as $7 \times 45 = 315$. Add larger remainder as $315 + 19 = 334$.

Answer is 334.

$334 = 19 \text{ mod } (45) \text{ \& } 15 \text{ mod } (29)$.

Problem 3: Find the number which yields 19 as the remainder when divided by 45, 15 as the remainder when divided as remainder when divided by 29. In addition to this, we will consider that this number yields 1 as the remainder when divided by 7.

Let us the same procedure for the first two combination and then answer will be,

$334 = 19 \text{ mod } (45) \text{ \& } 15 \text{ mod } (29)$.

Two consider the third divisor and remainder. We will consider that the number when divided by product of first two divisor will give 334.

That is, $N = 334 \text{ mod } (1305) \text{ \& } 1 \text{ mod } (7)$

Among, the above two, greater remainder is 334 and divisor corresponding to the greater remainder is 1305.

Difference between the remainder is $(334-1) = 333$.

Now, divide 1305 by 7, note down the quotient, 186 and remainder, 3.

Divide, 7 by 3, note down the quotient, 2 and remainder, 1.

Divide, 3 by 1, note down the quotient, 3 and remainder, 0.

Quotients: 186, 2, 3.

Number of quotients: 2 (even required, so remove first)

Let us consider optional number as 2.

2,3,2,333

2,339,2

680,339

Last two numbers are 680 & 339. Divide 680 by 7 (divisor of smaller remainder), we get quotient 97 & remainder 1. Multiple 1 with divisor of the greatest remainder as $1 \times 1305 = 1305$. Add larger remainder as $1305 + 334 = 1639$.

Answer is 1639.

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$1639 = 334 \bmod (1305) \text{ \& } 1 \bmod (7)$.

These problems to find numbers from divisor and remainder are important in following cases:

To find the revolution elapsed in a cycle, where we know the remainder of smaller cycles which are week cycle (7 days), lunar cycle, solar year cycle. Above method can be used to find the number of days elapsed since the start of the cycle (which is epoch).

6. Interesting thoughts

Mysterious forces and Nature's balance:

Perfection may lack happiness. Ultimate reason for our existence are may be due to imperfections. There are mysterious forces in universe which keeps nature in balance (can be imbalance also, but whatever we want). Example: Water has got some special properties which keeps our Earth and supported living beings in balance. Water cycle itself is known to all but surprising many mysteries are still within water. Water needs a specific amount of energy to get converted into gas from liquid. There are many benefits of water cycle, let us take one. One way of taking water from sea to clouds is by Sun. Water takes energy from Sun and gets converted to vapour. As it is converted into vapour and become LESS MASS, it is pushed up by the heavier gas above it. During this process of movement, vapour could have converted into water again but most of it remain vapour and forms clouds. When it condensed it releases Energy at the top of the atmosphere there by helping in one form in energy balance of Earth. Water and vapour state difference based on the weak hydrogen bond between one water molecule to the other. Such forces are there between any substance in the universe which are related to the energy added and released, distance between molecules and mass. Gravity is also similar force. Our brain signal network is also by similar force. Source of Energy, Mass and Gravity (forces) are the ultimate work of God and a mystery to us.

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Chapter V. Kalakriya Pada

Stanza 1:

वर्षम् द्वादशमासास् त्रिंशद्विषसस् भवेत्सस्मासस्तु ।
षष्टिस्राज्यस्दिवसस् षष्टिस्व विनाडिका नाडी ॥ १ ॥

“A year consist of 12 months. A month consists of 30days. A day consists of 60 nadis(नाडी). A nadi consists of 60 vinadikas(विनाडिका) or vinadis”

1 year = 12 months

1 month = 30 days

1 day = 60 nadis

1 nadi = 60 vinadikas

Stanza 2:

गुरुअक्षराणि षष्टिस्विनाडिका आर्क्षी षटेव वा प्राणास् ।
एवं कालविभागस्त्रविभागस्तथा भगणात् ॥ २ ॥

“A side real vinadika is equal to the time taken by a man in normal condition in pronouncing 60 long syllables with moderate flow of voice or in taking 6 respirations (pranas). This is the division of time. The division of a circle proceeds in a similar manner from the revolution”

So,

1 sidereal vinadika = 60 long syllables = 6 respiration (pranas)

Circular divisions similar to Time division

1 revolution = 12 signs

1 sign = 30 degrees

1 degree = 60 minutes (kalas)

1 minute = 60 seconds (vikalas)

1 second = 60 thirds (tatparas)

Stanza 3:

भगणास् द्वयोस् द्वयोस्ये विशेषशेषास्युगे द्वियोगास्ते ।
रविशशिनक्षत्रगणास्सम्मिश्रास्व व्यतीपातास् ॥ ३ ॥

“The difference between the revolution-numbers of any two planets is the number of conjunctions of those planets in a yuga. The combined revolutions of the Sun and the Moon added to themselves is the number of Vyatipatas in a Yuga”

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For example, let us consider that Object 1 is revolving around X at 1 revolution per minute & object 2 revolving at 2 revolution per minute. (Both at different radius). Let us also considering that they start their motion from aligned position (X-Obj1-Obj2 in straight line).

How many times they come in alignment in one minute without considering the starting and ending point?

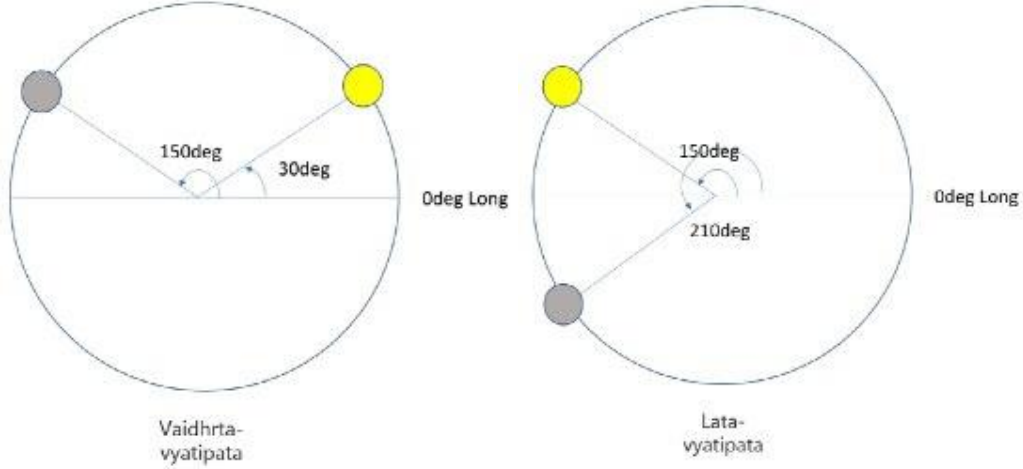
As per stanza (we have also seen this in relation motion between to objects in earlier chapter), that is difference between 2 & 1, $2-1 = 1$. So, once they come in alignment. If revolution number of Obj1 is 2rpm & Obj2 is 5rpm then they will come in alignment 3 times.

So, in a Yuga, number of conjunctions of two planets is the difference between their revolution numbers. (Here, time reference taken is in Yuga instead of a minute as in earlier examples). The phenomenon called Vyatipata is of two types: Lata-vyatipata & Vaidhrta-vyatipata. Vaidhrta-vyatipata occurs when the sum of the tropical longitudes of the Sun and the Moon amounts to 180degrees and Lata-vyatipata, when that sum amounts to 360deg. Thus, in one combined revolution of the Sun and the Moon there occur two vyatipatas.

In circle of asterism, Sun and Moon goes around Earth and position of Sun and Moon can be represented in terms of Longitudes by taking any position on Equator as reference (0deg Long). Now, at any point, position of Sun in the circle plot can be represented by its Longitude (A1) & similarly, Moon's position in the circle plot can be represented by its Longitude (A2). So, instance is said to be vyatipada when $A1 + A2$ is either 180deg or 360deg.

The conception of the phenomenon of vyatipada is very old. It occurs in the Vedanga-Jyoutisa, which states the number of vyatipatas in the yuga of 5 years. It also occurs in the Jaina astronomical work Jyotiskaronda, where the rule for finding the number of vyatipatas in a yuga of five years is formulated. These moments of two vyatipata are considered highly inauspicious.

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We will see vyatipada later in details with ksetra (circle of asterism) and longitudinal position of Sun and Moon in circle of asterism.

Stanza 4:

स्वौच्चभगणास्वभगणैस्विशेषितास्वौच्चनीचपरिवर्तास् ।
गुरुभगणास्राशिगुणासश्वयुजऽद्यासुरोरब्दास् ॥ ४ ॥

“The difference between the revolution-numbers of a planet and its ucca gives the revolutions of the planet’s epicycle in a yuga. The revolution-number of Jupiter multiplied by 12 gives the number of years beginning with Asvayuk in a yuga”

First line of the stanza is about Anomalistic and Synodic revolution of planets. We have already seen about Sidereal, Anomalistic and Synodic terms. Sidereal is number of revolutions with respect to fixed star, anomalistic is number of revolutions with respect to elliptic path points and Synodic is number of revolutions with respect to line joining Earth and Sun. Only the references are different & considering the variation of the reference, errors are calculated from other references.

The number of anomalistic revolutions of the Moon in a yuga, according to Aryabhata is

= Revolution number of the Moon - Revolution number of the Moon’s apogee (point on elliptic)

= 57753336 – 488219 (refer Gitika Pada)

= 57265117

The period of one anomalistic revolution of the Moon is equal to $1577917500/57265117 = 27.55459$ days or 27 days 13 hrs 18min 36 sec. Modern value is 27 days 13 hrs 18min 33 sec. Check the accuracy of value shared by

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Aryabhata, it is within 1 prana (4 sec). (1577917500 = 1582237500-4320000)
The number of synodic revolutions of the Moon in a yuga, according to Aryabhata is

= Revolution number of the Moon - Revolution number of the Sun (Geocentric)
= 57753336 – 4320000 (refer Gitika Pada)

= 53433336

The period of one synodic revolution of the Moon is equal to $1577917500/53433336$
= 29.5305818 days or 29days 12hrs 44min 2sec. Modern value is 29.53059 or 29days
12hrs 44min 3sec. Again, accuracy level within 1 prana (4 sec).
Similarly, Synodic revolutions & in days for other planets are given below with modern
value reference.

Mars – 2023176; 779.92125days; Modern value – 779.936days

Sighrocca of Mercury – 13617020; 115.8783days; Modern value – 115.877days

Jupiter – 3955776; 398.8895days; Modern value – 398.884days

Sighrocca of Venus – 2702388; 583.8975days; Modern value - 583.921days

Saturn – 4173436; 378.0859days; Modern value – 378.092days.

Second line of the stanza is about number of years in reference to Jupiter cycle.
Total number of revolutions of Sun in a yuga = 4320000
Total number of revolutions of Jupiter in a yuga = 364224
Number of years (geocentric) in a yuga = 4320000. To find number of years
(geocentric) per Jupiter revolution number, = $4320000/364224 = 11.860832$ (approx.
12).

Also, five 12 years cycles or 60 years forms the cycle of Years (Samvatsara). 60 years
are,

- | | | |
|-----------------|----------------|--------------------|
| 1. Vijaya, | 2. Jaya, | 3. Manmatha, |
| 4. Durumukha, | 5. Hemalamba, | 6. Vilamba, |
| 7. Vikari, | 8. Sarvari, | 9. Plava, |
| 10. Subhakrt, | 11. Sobhana, | 12. Krodhi, |
| 13. Visvavasv, | 14. Parabhava, | 15. Plavanga, |
| 16. Kilaka, | 17. Saumya, | 18. Sadharana, |
| 19. Virodhakrt, | 20. Paridhavi, | 21. Pramadi, |
| 22. Ananda, | 23. Raksasa, | 24. Nala or Anala, |
| 25. Pingala, | 26. Kalayukta, | 27. Siddhartha, |

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- | | | |
|-------------------|-----------------|----------------|
| 28. Raudra, | 29. Durmati, | 30. Dundubhi, |
| 31. Rudhirodgari, | 32. Raktaksa, | 33. Krodhana, |
| 34. Ksaya, | 35. Prabhava, | 36. Vibhava, |
| 37. Sukla, | 38. Pramoda, | 39. Prajapati, |
| 40. Angira, | 41. Srimukha, | 42. Bhava, |
| 43. Yuva, | 44. Dhata, | 45. Isvara, |
| 46. Bahudhanya, | 47. Pramathi, | 48. Vikrama, |
| 49. Vrsa, | 50. Citrabhanu, | 51. Subhanu, |
| 52. Tarana, | 53. Parthiva, | 54. Vyaya, |
| 55. Sarvajit, | 56. Sarvadhari, | 57. Virodhi, |
| 58. Vikrta, | 59. Khara, | 60. Nandana. |

There are specific meanings to above year names and there are reasons for assigning these names in this order.

A person when reaches 60 age, is celebrated as he has seen all samvatsara years starting from his birth year.

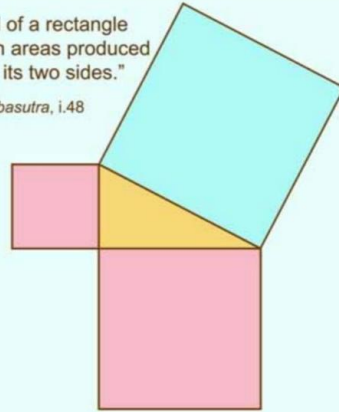
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27. Do you know?

- The *Shulbasûtras* give a precise geometric expression of the so-called "Pythagorean theorem."
- Right angles were made by ropes marked to give the triads 3, 4, 5 and 5, 12, 13 ($3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$).
- We should rename this theorem the "Shulba theorem"!

"The diagonal of a rectangle produces both areas produced separately by its two sides."

Baudhayana Sulbasutra, i.48

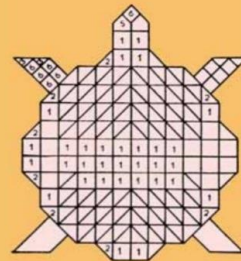


Examples of other geometric operations in the *Shulbasûtras*:

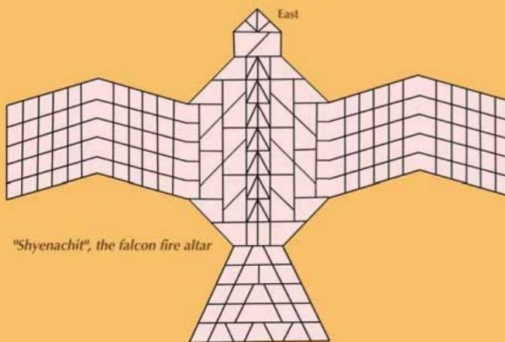
- "Squaring the circle" (and vice-versa): geometrically constructing a square having the same area as a given area.
- Adding or subtracting the areas of two squares (to produce a single square).
- Doubling the area of a square.
- In the last construction, $\sqrt{2}$ works out to 577/408 or 1.414215, correct to the 5th decimal. (Same precision with $\sqrt{3}$.)

28. Do you know?

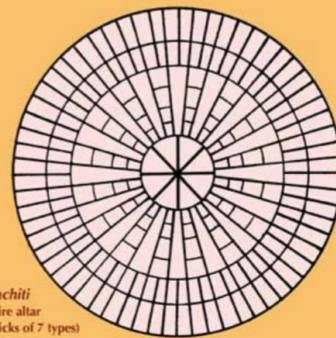
- Geometry of the *Shulbasûtras* (6th to 10th century BCE, possibly earlier): these four ancient texts deal with complex fire altars of various shapes constructed with bricks of specific shapes and area: the total area of the altar must always be carefully respected.
- This leads to precise but purely geometrical calculations (algebra does not exist yet).



Karmachit fire altar - first layer
(with 6 types of bricks totalling 200)



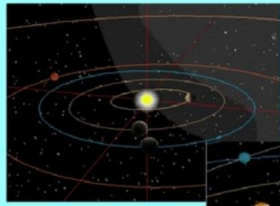
"Shyenachit", the falcon fire altar



The rathachakrachiti
or chariot-wheel fire altar
(first layer, 200 bricks of 7 types)

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29. Do you know?



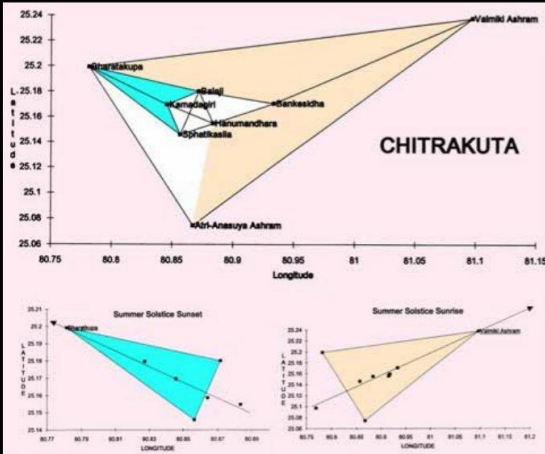
Āryabhata's “orbit of the sky”

- Āryabhata's “orbit of the sky” is 12,474,720,576,000 *yojanas* $\approx 16.8 \cdot 10^{13}$ km; therefore a diameter of $5.4 \cdot 10^{13}$ km, about 4,600 times our solar system's diameter.
- That is of the same order as 10 parsecs ($30 \cdot 10^{13}$ km), a distance where the Sun has a magnitude of 4.7, almost the limit of visibility to the human eye.

Bhaskara I: “For us, the sky extends to as far as it is illumined by the rays of the Sun. Beyond that, the sky is immeasurable.... The sky is beyond limit; it is impossible to state its measure.”

In other words, Āryabhata's “orbit of the sky” is of the same order as the distance “illumined by the sun”. Very likely a coincidence; nevertheless, his conception of the scale of the universe deserves to be noted.

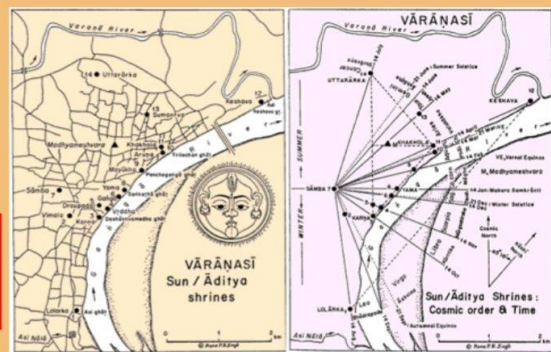
30. Do you know?



Chitrakut (associated with Lord Ram, who is often represented by a symbolic as an arrow): once mapped with GPS, ashrams form arrows that point to the sunrise and sunset on the summer solstice.

Varanasi: the 14 Aditya shrines precisely track the path of the sun through the year, month after month.

U.S. astrophysicist McKim Malville, with Indian scholars, studied India's sacred geography: at Varanasi, Chitrakut, Vijayanagara and other sites, sacred sites (shrines, ashrams etc.) were oriented towards specific points of the sun's path across the year.



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31. Do you know?

5. Unresolved Riddles

*"Swift and all beautiful art thou,
O Surya, maker of the light,
illuming all the radiant realm"*
(Rig-Veda, 1.50.4)

Sayana comments thus on the above hymn :

*tathā ca smaryate yojanānām
sahasre dve dve sate dve ca yojane
ekena nimisārdhena kramamāna*
"Thus it is remembered :
[O Surya] you who traverse
2,202 *yojana* in half a *nimesa*."

With a *yojana* of 13.6 km and a *nimesa* of $16/75^{\text{th}}$ of a second, this amounts to 280,755 km/s — just 6% from the speed of light (299,792 km/s): Coincidence? Intuition or inspired knowledge? Or some lost method of measurement? In any case, the fact should be noted.

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Stanza 5:

रविभगणास्रविअब्दास्रविशशियोगास् भवन्ति शशिमासास् ।

रविभूयोगास्दिवसास्भऽवर्तास्च अपि नाक्षत्रास् ॥ ५ ॥

“The revolutions of the Sun are Solar years. The conjunctions of the Sun and the Moon are lunar months. The conjunctions of the Sun and Earth are civil days. The rotations of the Earth are Sidereal days”

Refer, Gitika pada for number of revolutions.

As per the current stanza,

Solar years in a yuga = 4320000

Lunar months in a yuga = 53433336

Civil days in a yuga = 1577917500

Sidereal days in a yuga = 1582237500

Important things to note in the above stanza is that Aryabhata considered that the Earth revolves around the Sun and also the self-rotation of the Earth.

Stanza 6:

अधिमासकास्युगे ते रविमासेभ्यसधिकास्तु ये चान्द्रास् ।

शशिदिवसास्विज्ञेयास्भूदिवसऊनास्तिथिप्रलयास् ॥ ६ ॥

“The lunar months in a yuga which are in excess of the solar months in a yuga are known as the intercalary months in a yuga; and the lunar days in a yuga diminished by the civil days in a yuga are known as the omitted lunar days in a yuga”

As per first stanza of Kalakriya, a year has 12 months. So,

Number of solar months = 12 x 4320000

= 51840000

As per the current stanza,

Intercalary months in yuga = 53433336 – 51840000

= 1593336

As per first stanza of Kalakriya, a month has 30 days. So,

Number of lunar days in yuga = 30 x 53433336

= 1603000080

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As per the current stanza,

Omitted lunar days in a yuga = 1603000080 – 1577917500

= 25082580

Stanza 7:

रविवर्षं मानुष्यं ततपि त्रिंशत्गुणं भवति पित्र्यम् ।

पित्र्यं द्वादशगुणितं दिव्यं वर्षं विनिर्दिष्टम् ॥ ७ ॥

“A solar year is a year of men. Thirty times a year of men is a year of the manes. Twelve times a year of the Manes is called a divine year or a year of the gods”

As Manes reside in Moon & thirty lunar days is composed a lunar month, Aryabhata declared a day and night of Manes as 30 solar days of Earth. 12 times a year of Manes is called divine year, So,

1 divine year = 12 x year of Manes = 360 x year of Men

Stanza 8:

दिव्यं वर्षं सहस्रं ग्रहसामान्यं युगं द्विषट्कगुणम् ।

अष्टौत्तरं सहस्रं ब्राह्मस्दिवसस्रहयुगानाम् ॥ ८ ॥

“12000 divine years make a general planetary yuga. 1008 planetary yugas make a day of brahma”

As per Arybhata,

Yuga = 4320000 years

A day of brahma = 1008 yugas = 1008 x 12000 divine years

= 12096000 divine years

= 12096000 x 360 years of men

= 4354560000 years of men

Stanza 9:

उत्सर्पिणी युग अर्धं पश्चात्पसर्पिणी युग अर्धं च ।

मध्ये युगस्य सुषमा आदौ अन्ते दुष्षमा इन्दुउच्चात् ॥ ९ ॥

“The first half of a yuga is Utsarpini and the second half Apasarpini. Susama occurs in the middle and Dussama in the beginning and end. The time elapsed or to elapse is to be reckoned from the position of the Moon’s apogee”

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So, the yuga time cycle is divided in two halves (i) Utsarpini (auspicious half) (ii) Apasarpini (Avasarpini, inauspicious half). Yuga starts with Dussama, So the other cycle is Dussama-Susama-Dussama.

Last point is on taking Moon's apogee for calculating the elapsed time and to elapse. This is because the abraded yuga and corresponding revolution of Moon's apogee is the same as that of number of days in yuga and its own revolution number. Let us see abraded yuga (in days) and corresponding revolution number.

Sun: $u = 210389$; $v = 576$

Moon: $u = 2155625$; $v = 78898$

Moon's apogee: $u = 1577917500$; $v = 488219$

Moon's ascending node: $u = 788958750$; $v = 116113$

Mars: $u = 131493125$; $v = 191402$

Sighrocca of Mercury: $u = 78895875$; $v = 896851$

Jupiter: $u = 131493125$; $v = 30352$

Sighrocca of Venus: $u = 131493125$; $v = 585199$

Saturn: $u = 394479375$; $v = 36641$

7. Interesting thoughts

Gravity:

What would happen if Earth's gravity increases very fast that it compresses itself infinitely?

Earth->Liquids->Gaseous->Plasma->Electron

Degeneration->Proton

Degeneration->Neutron

Degeneration->Quark

Degeneration->Preon

Degeneration->Singularity

Stanza 10:

षष्टिअब्दानां षष्टिस्यदा व्यतीतास्त्रयस्च युगपादास् ।
त्रिअधिका विंशतिसब्दास्तदा इह मम जन्मनसतीतास् ॥ १० ॥

"When sixty times sixty years and three quarter yugas had elapsed, twenty-three years had then passed since my birth"

As per first line, 3600 years had elapsed since the beginning of the current kaliyuga. This epoch corresponds to mean noon at Ujjayini, Sunday, March 21, 499AD. At this time Aryabhata was exactly 23 years of age. Aryabhata was born on March 21, 476AD. Kali epoch can be found based on this date or it can also be obtained by observation on planets and Moon's apogee by Kuttaka method. Next stanza gives reference to the start of the yuga.

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We have already discussed on this in first chapter.

Stanza 11:

युगवर्षमासदिवसास्समं प्रवृत्तास्तु चैत्रशुक्लऽदेस् ।
कालसयं अनादिअन्तस्ग्रहभैस् अनुमीयते क्षेत्रे ॥ ११ ॥

“The yuga, the year, the month, and the day commenced simultaneously at the beginning of the light half of Caitra. This time, which is without beginning and end, is measured with the help of planets and the asterisms on the celestial sphere”

As mentioned by Aryabhata, Time is endless and the beginning considered here is only for the purpose of measurement as it needs a beginning reference of us to start the time. So, a beginning is considered by observation when Caitra star which means at the beginning of light fortnight of Caitra when half the Sun had risen above the horizon at Lanka.

So, Aryabhata mentions, time is endless in a loop of Yugas. When a yuga completes then next yuga will start at the same beginning and same will continue endlessly. As mentioned by Aryabhata, the start of yuga is defined by the position of the planets and asterisms. The calculation of the positions of the planets is ultimately aimed at the determination of time, hence the stanza. So, beginning of yuga is defined by the star Caitra, alignment/ position of planets and position of stars on the celestial sphere.

Stanza 12:

षष्ठ्या सूर्याब्दानां प्रपूरयन्ति ग्रहास्मपरिणाहम् ।
दिव्येन नभस्परिधिं समं भ्रमन्तस्स्वकक्ष्यासु ॥ १२ ॥

“The planets moving with equal linear velocity in their own orbits complete a distance equal to the circumference of the sphere of the asterisms in a period of 60 solar years and a distance equal to the circumference of the sphere of the sky in a yuga”

As per Aryabhata, a planet moves through a distance of 173260008 yojana in 60 solar years and a distance of 12474720576000 yojanas in 4320000 solar years. Since there are 1577917500 civil days in 43200000 years, it follows that the mean daily motion of a planet, according to Aryabhata is $(12474720576000/1577917500)$ yojanas or approx. 7905.8 yojanas.

Stanza 13:

मण्डलं अल्पं अधस्तात्कालेन अल्पेन पूरयति चन्द्रस् ।
उपरिष्ठात्सर्वेषां महत्त्व महता शनैश्चारी ॥ १३ ॥

“Moon completed its lowest and smallest orbit in the shortest time; Saturn completes its highest orbit in the longest time”

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We have already seen the length of orbit in Githika pada Chapter III stanza 6. Calculating that method, we can find the orbit of all planets, Moon, stars and Sun.
Orbit of the sky = $57753336 \times 12 \times 30 \times 60 \times 10$ yojanas

$$= 12474720576000 \text{ yojanas.}$$

$$\text{Orbit of the Sun} = 12474720576000/4320000$$

$$= 2887666 \text{ and } 4/5 \text{ yojanas.}$$

$$\text{Orbit of the Moon} = 216000 \text{ yojanas. (Smallest)}$$

$$\text{Orbit of Mars} = 5431291 \text{ and } 132027/287103 \text{ yojanas}$$

$$\text{Orbit of sikhrocca of Mercury} = 695473 \text{ and } 373277/896851 \text{ yojanas}$$

$$\text{Orbit of Jupiter} = 35250133 \text{ and } 699/1897 \text{ yojanas}$$

$$\text{Orbit of sikhrocca of Venus} = 1776421 \text{ and } 255221/585199 \text{ yojanas}$$

$$\text{Orbit of Saturn} = 85114493 \text{ and } 5987/56641 \text{ yojanas (Longest)}$$

$$\text{Orbit of asterisms} = 173260008 \text{ yojanas.}$$

Stanza 14:

अल्पे हि मण्डले अल्पा महति महान्तस्च राशयस्त्रेयास् ।
अंशास्कुलास्तथा एवं विभागतुल्यास्वकक्ष्यासु ॥ १४ ॥

“The linear measures of the signs are to be known to be small in small orbits and large in large orbits; so, also are the linear measures of the degrees, minutes, etc., the circular division is however the same in the orbits of the various planets”

In comparison with the earlier stanza, we can convert the length of the circular division of the orbits of different planets (this is with the assumption that they follow circular orbit, however, it is actually elliptical with offset).

Moon: 1 minute – 10 yojanas.

Sikhrocca of Mercury: 1 minute – 32 yojanas.

Sikhrocca of Venus: 1 minute – 82 yojanas.

Sun: 1 minute – 134 yojanas.

Mars: 1 minute – 251 yojanas.

Jupiter: 1 minute – 1586 yojanas.

Saturn: 3940 yojanas.

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As per the second line, the angles are the same but angular velocities are different.
Following are the angular velocities of different planets:

Sun: 59'8" per day;

Moon: 590'35" per day;

Moon's apogee: 6'41" per day;

Moon's ascending node: 3'11" per day;

Mars: 31'26" per day;

Signrocca of Mercury: 4deg 5' 32" per day;

Jupiter: 4' 59"per day;

Sighrocca of Venus: 1deg 36' 8" per day;

Saturn: 2'0" per day.

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32. Do you know?

Testimonies from two French mathematicians:

- “The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of Antiquity, Archimedes and Apollonius.” — Laplace (early 19th century)
- “The Indian mind has always had for calculations and the handling of numbers an extraordinary inclination, ease and power, such as no other civilization in history ever possessed to the same degree. So much so that Indian culture regarded the science of numbers as the noblest of its arts.... A thousand years ahead of Europeans, Indian savants knew that the zero and infinity were mutually inverse notions....” — Georges Ifrah (1994)

33. Do you know?

“A thousand years ahead of Europeans,
Indian savants knew that the zero and
infinity were mutually inverse notions....”
— Georges Ifrah

Khachheda means “divided by *kha*”;
Kha (space) stands for zero;
“Divided by zero” = infinity.

— Brahmagupta,
Brahmasphuta Siddhanta (628 CE)

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34. Do you know?

India was a pioneer in many technologies.

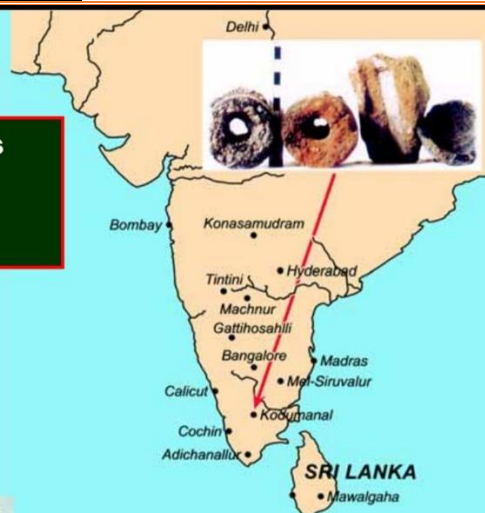


- Metallurgy (bronze, iron, wootz, zinc...)
- Pottery (ceramic, faience...)
- Pigments (painting, dyeing...)

- Perfumes & cosmetics
- Medicines
- Chemistry and alchemy

35. Do you know?

Sites where wootz steel was prepared (note Kodumanal near Coimbatore).



The Delhi Iron Pillar: a thin layer of iron and phosphorus compound makes it rustproof, even after more than 1500 years.

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Stanza 15:

भानां अधश्चनैश्चरसुरगुरुभौमार्कशुक्रबुधचन्द्रास् ।
एषां अधस्व भूमिस्मेधीभूता खमध्यस्था ॥ १५ ॥

“The asterisms are the outermost. Beneath the asterism lie the planets Saturn, Jupiter, Mars, the Sun, Venus, Mercury and the Moon one below the other; and beneath then all lies the Earth like the hitching pug in the midst of space”

This stanza explains the order of planets positioned in solar system considering geocentric approach. Considering the Aryabhata shared sikhrocca of Mercury and Venus, it is clear that he considered Mercury and Venus revolving the Sun inbetween the Earth and the Sun, the way Moon does the Earth.

Stanza 16:

सप्त एते होराईशास्त्रनैश्चरऽद्यास्यथाक्रमं शीघ्रास् ।
शीघ्रक्रमात् चतुर्थास् भवन्ति सूर्योदयादिनपास् ॥ १६ ॥

“The above mentioned seven planets beginning with Saturn, which are arranged in the order of increasing velocity, are the lords of the successive hours. The planets occurring fourth in the order of increasing velocity are the lords of the successive days, which are reckoned from sunrise at Lanka”

That is to say, the lords of the hours are Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Saturn, Jupiter,....

And the lords of the seven days cycle (week)
Saturn, Sun, Moon, Mars, Mercury, Jupiter, Venus, Saturn, Sun,...

It can be noted that as per Aryabhata, the first day of the week start from Saturday.

Stanza 17:

कक्ष्याप्रतिमण्डलगास् भ्रमन्ति सर्वे ग्रहास्वचारेण ।
मन्दौच्चातनुलोमं प्रतिलोमं च एव शीघ्रौच्चात् ॥ १७ ॥

“All the planets move by their mean motion on their orbits and their eccentric circles from the apsis eastward and from the conjunction westwards”

The mean planet moves with its mean motion on its orbit the center of which is the center of the Earth. The true planet moves with its mean motion on an eccentric circle (elliptic path) the center of which does not coincide with the center of the Earth. Kaksya or Kaksyamandala means the orbit on which the mean planet moves. Pratimandala means eccentric circle on which the true planet moves. Because of the eccentricity of this second circle the planet is sometimes seen ahead of and sometimes back of its mean place.

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Stanza 18:

कक्ष्यामण्डलतुल्यं स्वं स्वं प्रतिमण्डलं भवति एषाम् ।

प्रतिमण्डलस्य मध्यं घनभूमध्याततिक्रान्तम् ॥ १८ ॥

“The eccentric circle of each planet is equal to its kaksyamandala (the orbit on which the mean planet moves). The center of the eccentric circle is outside the center of the solid Earth”

Sharing orbital length of planets from Gitika pada for reference.

Orbit of the sky = $57753336 \times 12 \times 30 \times 60 \times 10$ yojanas

= 12474720576000 yojanas.

Orbit of the Sun = $12474720576000/4320000$

= 2887666 and $4/5$ yojanas.

Orbit of the Moon = 216000 yojanas. (Smallest)

Orbit of Mars = 5431291 and $132027/287103$ yojanas

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Orbit of sighthrocca of Venus = 1776421 and $255221/585199$ yojanas

Orbit of Saturn = 85114493 and $5987/56641$ yojanas (Longest)

Orbit of asterisms = 173260008 yojanas.

Stanza 19:

प्रतिमण्डलभूविवरं व्यास अर्धं स्वौच्चनीचवृत्तस्य ।

वृत्तपरिधौ ग्रहास्ते मध्यमचारात् भ्रमन्ति एवम् ॥ १९ ॥

“The distance between the centre of the Earth and the centre of the eccentric circle is equal to the semi-diameter of the epicycle of the planet. All the planets undoubtedly move with mean motion on the circumference of the epicycle”

At the end of this chapter, we will discuss on the epicycle model of planetary motion shared by Aryabhata.

Stanza 20:

यस्शीघ्रगतिस्वौच्चात्प्रतिलोमगतिस्त्वृत्तकक्षायाम् ।

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अनुलोमगतिस्वृत्ते मन्दगतिस्प्रहस् भवति ॥ २० ॥

“A planet when faster than its ucca moves clockwise on the circumference of its epicycle and when slower than its ucca moves anticlockwise on its epicycle”

The planet moves on its epicycle pratiloma from its sighthrocca and anuloma from its mandocca. On the epicycle of the apsis the motion should be exactly the reverse of these.

Anuloma means “ahead” and pratiloma means “behind”. So, Anuloma and Pratiloma refers to the planets position with reference to the mean planet as ahead of it or behind it. During the half of a planet’s revolution on its epicycle the planet is ahead of the mean planet and during half of its revolution is behind the mean planet. We will see method of finding the position of planets in next stanza.

Stanza 21:

अनुलोमगानि मन्दात्शीघ्रात्प्रतिलोमगानि वृत्तानि ।

कक्ष्यामण्डललग्नस्ववृत्तमध्ये ग्रहस्मध्यस् ॥ २१ ॥

“The epicycle move anticlockwise from the apogees and clockwise from the sighthroccas. The mean planet lies at the centre of its epicycle, which is situated on the planet’s orbit”

Stanza 22:

क्षयधनधनक्षयास् स्युर्मन्दौच्चात्त्ययेन शीघ्रौच्चात् ।

शनिगुरुकुजेषु मन्दात् अर्ध ऋणं धनं भवति पूर्वे ॥ २२ ॥

“The correction from the apogee for the four anomalistic quadrants are respectively minus, plus, plus and minus. Those from the sighthrocca are just the reverse. In the case of the superior planets Saturn, Jupiter and Mars, first apply the mandaphala negatively or positively as the case may be”

First point is on the sign convention based on the quadrant which is similar to our Sine and Cosine convention.

Correction from the apogee = -correction for the I quadrant +correction for the II quadrant +correction for the III quadrant – correction for the IV quadrant

Correction from the sighthrocca = +correction for the I quadrant -correction for the II quadrant -correction for the III quadrant + correction for the IV quadrant

In general, the above formula combine with the method explained in Gitika pada can be combined to calculate mandaphala & sighraphala corrections.

Mandaphala = $R \sin(\text{planet's mandakendra bhuja}) \times \text{manda epicycle}/360$

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Sighraphala = $R \sin(\text{planet's sigrakendra bhuja}) \times \text{sighra epicycle} \times R / (360 \times H)$

H is planets sigrakarna.

We will geometric representation of the above in coming days.

Stanza 23:

मन्दौच्चात्शीघ्रौच्चात् अर्धं ऋणं धनं ग्रहेषु मन्देषु ।

मन्दौच्चात्स्फुटमध्यात्शीघ्रौच्चात् स्फुटास्त्रेयास् ॥ २३ ॥

“Apply half the mandaphala and half the sighraphala to the planet and to the planet’s apogee negatively or positively. The mean planet then corrected for the mandaphala calculated again from the new mandakendra is called the true-mean planet and that true mean planet corrected for the sighraphala calculated again is known as the true planet”

As per the above stanza, for calculation true position (true longitude) of the planet, first apply half of the mandaphala to the mean longitude of the planet negatively or positively, mandakendra is less than or greater than 18deg. Same way to the longitude of the planet’s apogee in reverse. Then, apply half of sighraphala to the corrected longitude of the planet’s apogee negatively or positively, sigrakendra is less than or greater than 180deg.

From the corrected position of the planet, do the above again to get true position of planet.

Stanza 24:

शीघ्रौच्चात् अर्धऊनं कर्तव्यं ऋणं धनं स्वमन्दौच्चे ।

स्फुटमध्यौ तु भृगुबुधौ सिद्धात्मन्दात्स्फुटौ भवतस् ॥ २४ ॥

“In the case of Mercury and Venus apply half the sighraphala negatively or positively to the longitude of the planet’s apogee according as the sigrakendra is less than or greater than 180deg. From the corrected longitude of the planet’s apogee, calculate the mandaphala afresh and apply it to the mean longitude of the planet; then are obtained the true-mean longitudes of Mercury and Venus. The sighraphala calculated afresh, having applied to then they become true longitudes.”

Stanza 25:

भूताराग्रहविवरं व्यास अर्धहतस्वकर्णसंवर्गस् ।

कक्षयायां ग्रहवेगस्यस् भवति सस्मन्दनीचौच्चे ॥ २५ ॥

“The product of the mandakarna and the sigrakarna when divided by the radius gives the distance between the Earth and the planet. The velocity of the true planet moving

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on the sikhra epicycle is the same as the velocity of the true mean planet moving in its orbit of radius equal to the mandakarna”

That it, the distance between the Earth and a planet is equal to (mandakarna x sikhra) / R.

With this Kalakriya is over. We will see geometric representation of some stanza of kalakriya for better understanding. Some part of which is also required to understand next chapter, Golapada.

To calculate true planetary position on a particular day at sunrise for any place, there are few corrections required to be applied on mean planetary position. Correction are of two kinds (1) manda correction (2) sikhra correction. For the Sun and Moon, the first correction alone suffices to calculate the true position. For other planets (Mercury, Venus, Mars, Jupiter and Saturn), both corrections are necessary. Reason is simple, Moon revolves around the Earth and Earth being the focus of revolution. As, Earth revolves about itself, Sun relatively revolves around Earth and Earth being relative focus of revolution. However, for other planets, Earth is not the center focus of revolution and needs an additional correction for calculating true position by geocentric approach.

By first approximation, the mean planetary positions signify that we have presumed that the planets are going in a circle. The manda-karna is therefore intended to get the corresponding position in the ellipse from the position in the circle. There are two concepts of (i) Eccentric theory & (ii) Epicyclic theory. First, we will see Eccentric theory.

Eccentric theory is stated in stanza 18 of kalakriya pada. “The eccentric circle of each planet is equal to its kaksyamandala (the orbit on which the mean planet moves). The center of the eccentric circle is outside the center of the solid Earth”. Since the center of the eccentric circle is outside the center of the Earth with an eccentricity, Bhujaphalam, Equation of centre is considered in case of mandaphala. This is used as correction required for planets to convert their mean position to true position.

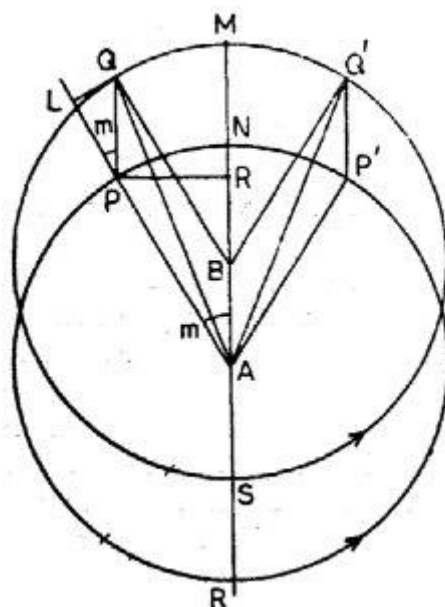
Let A be the center of the Earth. Let P be the mean planet which goes round in a circle whose center is A. Let B be the center of the circle, in which the planet actually moves with radius same as the earlier circle.

The circle A is the mean orbit, whereas the circle B is the true orbit. The mean planet moves on the mean orbit called as the deferent. The position of the true planet in the eccentric circle B namely Q will be such that PQ is parallel and equal to AB. Here, AP = BQ. Also, we can see that when B is vertically above A and when the radii of the two circles are equal, every corresponding point of the circle A will be vertically moved by the same distance AB. Join AQ to cut the circle A in T, which is said to represent the true planet. Evidently when the planet is at N in the mean orbit, the planet is at M, in the eccentric, which point is called the mandocca.

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When the planet is at R in the mean orbit the planet is at S in the eccentric. Please note that these points M & S corresponds to the apogee and perigee in the case of the mandaphala correction, i.e., equation of the center.

When the planet is at M in the eccentric, the position of the true planet coincides with N, the mean planet, so that the mandaphala correction is zero. Similarly, the true planet at R being in the same direction as AS, there also the mandaphala is zero. When the planet is at P, since the true planet is at T, the correction mandaphala PT is negative. When the mean planet is at P' on the right, the true planet will be at T' so that correction P'T' is positive. The angle NAP is called mandakendra which corresponds to Mean anomaly of modern astronomy. So, when mean anomaly is between 0deg to 180deg then mandaphala (equation of center) is negative. At 0deg and 180deg (apogee and perigee), mandaphala is zero. When mean anomaly is between 180deg to 360deg, mandaphala is positive.



In case of sighraphala, to get the geocentric planet from the heliocentric, we construe the longitude of the sighrocca minus the longitude of the planet as the kendra. The arc PT is approximately taken to be equal to QL the perpendicular dropped from Q on AP produced. This approximation can be considered as the difference will not be much in the case of the equation of centre. From similar triangle stanza 15 of Ganita pada.

$$QL / PQ = PK / AP = \sin m$$

$$QL = PQ \times \sin m = PT \text{ (approx..)} = \text{mandaphala or equation of center}$$

Here, in modern astronomy, same is

$$2e \times \sin m + (5/4) \times e^2 \times \sin (2m)$$

Which is approximately,

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$$PQ \times \sin m = 2e \times \sin m$$

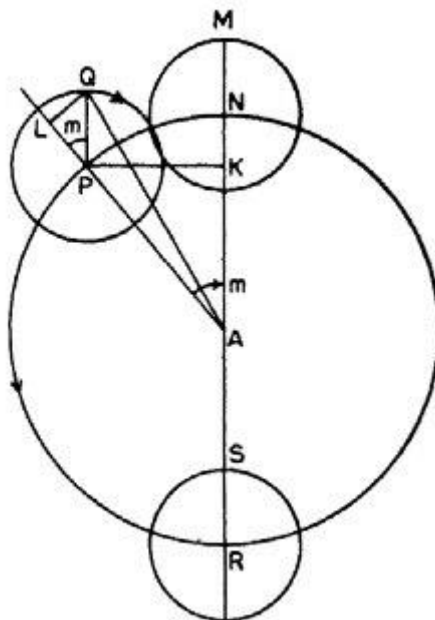
$$PQ = 2e$$

which is called antyaphala (maximum equation of center).

In Epicyclic theory, the circle A is the mean orbit. Let P be the mean planetary position. The small circle as shown in figure is called the epicycle. The position of the true planet Q is always taken to be such that PQ is parallel to AN. The length of PQ is equal to the length of AB, the distance between the two centers of the mean orbit and the true orbit, both eccentric and epicyclic theory give the same position of the true planet.

$$QL = PQ \times \sin m.$$

Aryabhata gave the value of circumference of the epicycle of planets instead of PQ, radius of epicycle. Also, these circumferences are shared in degree by Indian astronomers which will help in getting the equation of centre in degree.



As already mentioned, sigrhaphala is applicable only for the planets Mars, Mercury, Jupiter, Venus and Saturn. This is because they revolve not around the Earth, but round the Sun. Hence this sigrhaphala is for converting heliocentric position to geocentric.

Now, in case of the equation of centre, considering the arc PT as roughly equal to QL (because the equation of time is small). In case of sigrhaphala, this is larger, so we have to find this value. Here,

EQ is called Sighrakarna.

$$K^2 = EQ^2 \times (EP+PL)^2 + QL^2$$

$$K^2 = EN^2 + QN^2$$

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Here, K is EQ, EP = R (triya), PQ is antyaphala or maximum sighraphala, QL is bhujaphala & PL is kotiphala, QN = PK called bhuja, EK is koti. So,

$$K^2 = (\text{triya} + \text{kotiphala})^2 + \text{bhujaphala}^2$$

$$= R^2 + a^2 \pm 2R \times \text{kotiphala}$$

$$K^2 = (\text{koti} + \text{bhujaphala})^2 + \text{bhuja}^2$$

$$= R^2 + a^2 \pm 2a \times \text{koti}$$

Also, $PT/LQ = ET/EQ$ (approximately)

$$PT = ET \times LQ / EQ$$

$$= R \times LQ / K$$

But, $LQ/PK = PQ/EP$

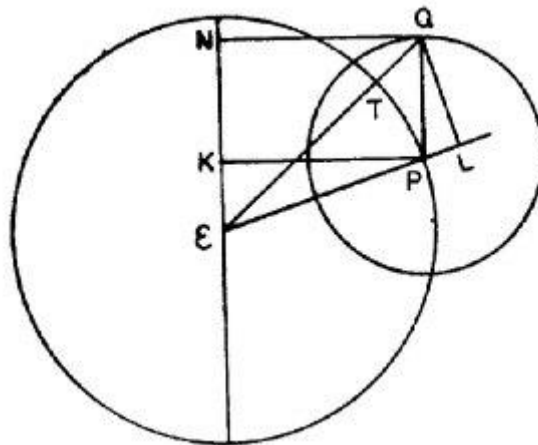
So,

$$PT = R \times PQ \times PK / (K \times R)$$

$$= a \times PK / K$$

$$= H \sin m \text{ (Hindu Sine)}$$

Here, m is called sighra anomaly, PT = Sighraphala = $a \sin m$, K is sighrakarna.



8. Interesting thoughts

What is this Singularity? Properties of this Singularity? Why a name Singularity? Can the above process reverse from Singularity?

Can you prove that total mass of all substance in the universe at any given time is constant (with the help of laws that govern substance)?

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Chapter VI. Gola Pada

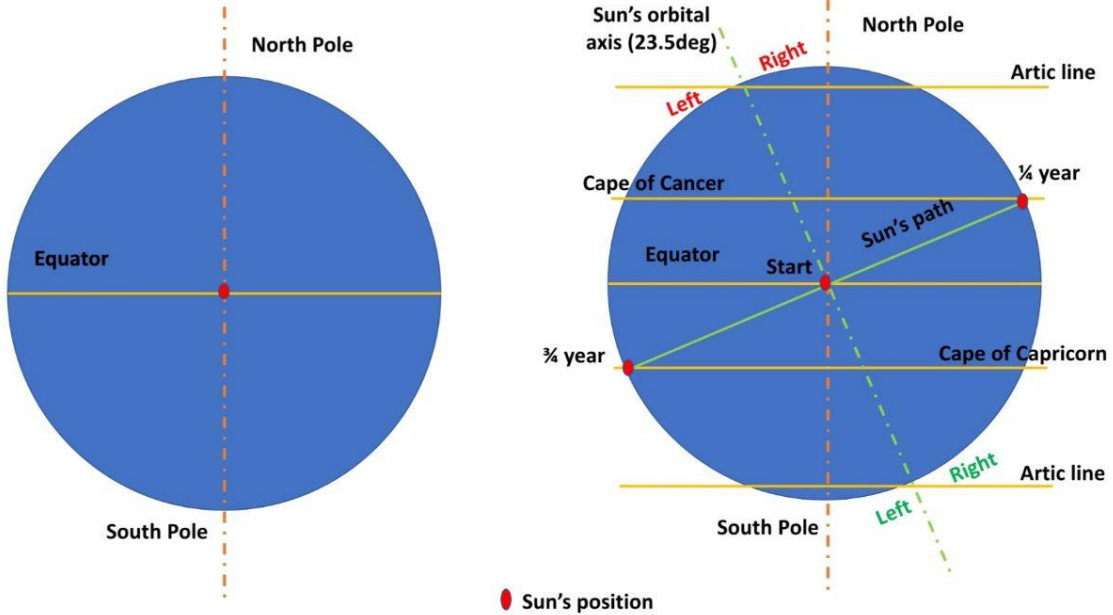
Stanza 1:

मेषऽदेस्कन्याअन्तं समं उदचपमण्डल अर्ध अपयातम् ।

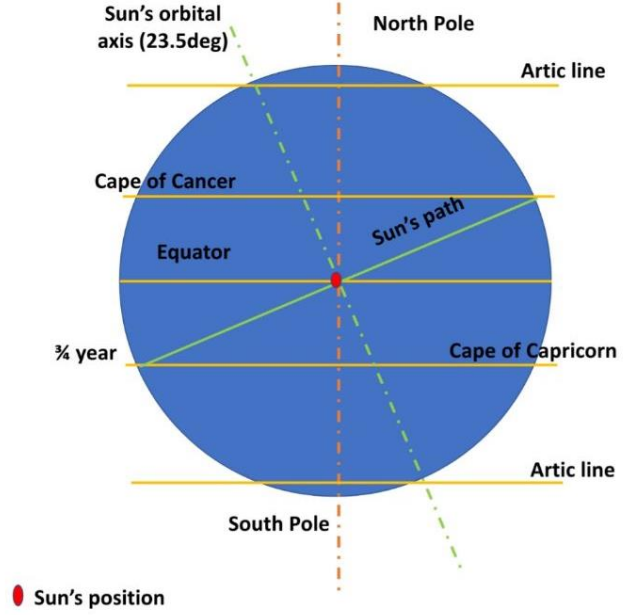
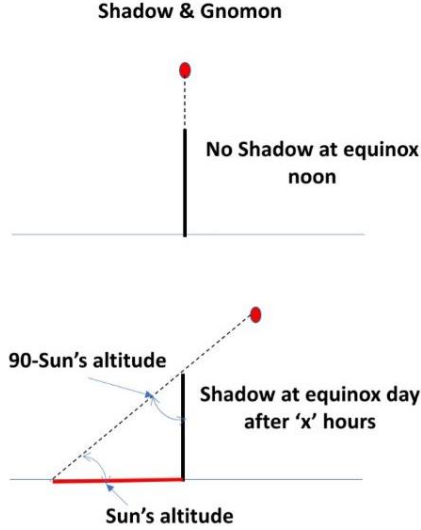
तौल्यऽदेस्मीनान्तं शेष अर्ध दक्षिणेन एव ॥ १ ॥

“One half of the ecliptic, running from the beginning of the sign Aries to the end of the sign Virgo, lies obliquely inclined to the equator northwards. The remaining half of the ecliptic running from the beginning of the sign Libra to the end of the sign Pisces, lies equally inclined to the equator southwards”

This shows the movement of the Sun with respect to the obliquity towards Northern and Southern hemisphere. It is observed with respect to the distant star. It begins with star Aries to Virgo in North of equator. It begins with star Libra to Pisces in South of equator.



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Stanza 2:

ताराग्रहैन्दुपातास् भ्रमन्ति अजस्रं अपमण्डले अर्कस्व ।

अर्कात्त्व मण्डल अर्धे भ्रमति हि तस्मिन्क्षितिछाया ॥ २ ॥

“The Sun, the nodes of the planets, and the node of the Moon move constantly along the ecliptic. The shadow of the Earth moves along the ecliptic at a distance of 180degrees from the Sun”

It is simple, like a ball placed in front of light and the rear side of the ball which is half the ball will be dark due to the shadow of the ball. Same way, the shadow of the Earth moves at a distance half a circle and moves along the ecliptic as the Sun moves in ecliptic relatively to Earth.

Nodes of the planets and Moon also moves constantly along the path of the ecliptic. Planets and the Moon intersect the node ecliptic at two points, ascending node and descending node.

Stanza 3:

अपमण्डलस्य चन्द्रस्पातात् याति उत्तरेण दक्षिणतस् ।

कुजगुरुकोणास्च एवं शीघ्रौच्चेन अपि बुधशुक्रौ ॥ ३ ॥

“The Moon moves to the north and to the south of the ecliptic from its nodes. So also do the planets Mars, Jupiter and Saturn. Similar is also the motion of the sikhroccas of Mercury and Venus”

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As per Aryabhata, Mercury and Venus revolve round the Sun with the velocity of their sighthroccas and so the latitudes of Mercury and Venus are really the latitudes of their sighthroccas.

For Moon,

$R \times \sin(\text{latitude}) = R \times \sin(\text{true longitude of Moon} - \text{True longitudes of ascending node}) \times R \times \sin(\text{inclination of orbit to the ecliptic}) / \text{Moon's true distance or mandakarna}$

For Mars, Jupiter and Saturn,

$R \times \sin(\text{latitude}) = R \times \sin(\text{true longitudes of the planet} - \text{true longitude of ascending node}) \times R \times \sin(\text{inclination of the ecliptic}) / \text{Distance of the planet form the Earth}$

(refer Kalakriya pada stanza 25)

For Mercury and Venus,

$R \times \sin(\text{latitude}) = R \times \sin(\text{true longitude of planet} - \text{true longitude of ascending node}) \times R \times \sin(\text{inclination of the planet to the ecliptic}) / \text{distance of planet from the Earth}$

Stanza 4:

चन्द्रसंशैस् द्वादशभिसविक्षिप्तसर्कान्तरस्थितस्दृश्यस् ।

नवभिस्मृगुस्मृगोस्तैस् द्विअधिकैस् द्विअधिकैस्यथा श्लक्षणास् ॥ ४ ॥

“When the Moon has no latitude, it is visible when situated at a distance of 12 degrees from the Sun. Venus is visible when 9 degrees distance from the Sun. The other planets taken in order of decreasing sizes viz., Jupiter, Mercury, Saturn and Mars are visible when they are 11, 13, 15 and 17 degrees of time distant from the Sun”

One degree of time is equivalent to 4 minutes. When the moon is ahead of the Sun, is visible towards the west if the arc of the ecliptic joining the Sun and the Moon takes atleast 12 x 4 (48 minutes) in setting below the horizon and when behind the Sun, it is visible towards the east if the arc of the ecliptic joining the Sun and the Moon takes at least 12x4 (48) minutes in rising above the horizon. The Moon will be visible at a place if the time interval between Sunrise and Moonrise or between Sunset and Moonset, amounts to 12 x 4 minutes or more. This is applicable if the Moon has no latitude. Above stanza explains the waxing of Moon. It can be seen that we get to see Full moon when the moon rise and sun rise is separated further apart. Also note that due to declination of Moon orbital axis, we get to see full moon as we(Earth) don't block the light from Sun onto Moon. As per Aryabhata, source of light is only Sun and all the other planets and Moon only reflect the Sun light.

Also, we get to see a Lunar eclipse when the Sun rise and Moon rise is farther part and they meet in their orbital path that is at the node. Same applies to other the planets visibility also. It is to be noted that visibility of the

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planets and Moon are related to Sunrise and Sunset (position of Sun) as they are only reflecting the Sun's light.

If they have their own light then Sun's position should not matter to see them from Earth even if Earth's shadow falls on them.

Stanza 5:

भूग्रहभानां गोल अर्धानि स्वछायया विवर्णानि ।

अर्धानि यथासारं सूर्याभिमुखानि दीप्यन्ते ॥ ५ ॥

"Halves of the globes of the Earth, the planets and the stars are dark due to their own shadows; the other halves facing the Sun are bright in proportion to their sizes"

Aryabhata, in this stanza, clearly brings out that the Sun is the source of light for the brightness of the planets, Earth and even the stars. His reference to stars and their brightness is not correct. It is to be noted that he refers to the shape of Earth and planets as spherical.

Next lines, on the rear portion of the Earth and planet, the night time, darkness due to their shadows of their own.

Stanza 6:

वृत्तभपञ्जरमध्ये कक्ष्यापरिवेष्टितस्वमध्यगतस् ।

मृदजलशिखिवायुमयम्भूगोलस्सर्वतस्वृत्तस् ॥ ६ ॥

"The globe of the Earth stands in space at the centre of the circular frame of the asterism surrounded by the orbits; it is made up of water, earth, fire and air and is spherical"

Pupil of Aryabhata, Somesvara says that, as per Aryabhata, "the Earth, mother of all beings, stands motionless in space" is against the teachings of Aryabhata and also about the information shared in earlier chapters. However, he concludes in considering this for deriving the position and motion of other planets. This stanza is put forth to consider geocentric concept to arrive at the position of the planets, Moon and Sun. Aryabhata referred to the shape of Earth as spherical numerous time in his work and also he refers that Earth is made of water, Earth (solid masses), fire and air.

Stanza 7:

यद्वत्कदम्बपुष्पग्रन्थिस्प्रचितस्समन्ततस्कुसुमैस् ।

तद्वत्हि सर्वसत्त्वैस्जलजैस्स्थलजैश्च भूगोलस् ॥ ७ ॥

"As the bulb of kadamba flower is covered all around with blossoms, the globe of the Earth is covered by creatures terrestrial and aquatics"

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This stanza is about the outer layer which covers the Spherical Earth. We live in the surface of the Earth which is like a globe shaped spherical kadamba flower with so many blossoms around it.

Stanza 8:

ब्रह्मदिवसेन भूमेसुपरिष्ठात्योजनं भवति वृद्धिस् ।

दिनतुल्यया एकरात्र्या मृदुपचिताया भवति हानिस् ॥ ८ ॥

"During a day of Brahma, the size of Earth increases externally by one yojana; and during a night of Brahma which is as long as the day, this growth of the Earth is destroyed"

As per geologist, the rate of the increase in size varies as per location on Earth and time. Minimum rate of uplift at Himalayas is about 6in per century, whereas in Greenland it is about 3mm per year.

Also, astronomers believe that size of Earth can also increase by very little amount by the bombardment of meteorites on Earth.

This upliftment is due to the tectonic plate movement which is very slow (at approximately equal to the growth of our finger nail) but significant over many years. They are one of the reason for Earth quakes. When these plates slide over another, they compress the below plate and also the top plate climb over bottom. This will create local upliftment. Force acting on these plates for their movement is so high that it lifts the plate rubbing over another and overcoming gravity force acting downwards towards the center of mass of Earth.

Stanza 9:

अनुलोमगतिस्त्रौस्थस् पश्यति अचलं विलोमगं यद्वत् ।

अचलानि भानि तद्वत्समपश्चिमगानि लङ्कायाम् ॥ ९ ॥

"Just as a man in a boat moving forward sees the stationary object on either side of the river as moving backwards, just so are the stationary stars seen by people at lanka moving exactly towards the west"

This stanza is about the apparent relative motion of star due to Earth's rotation. It can be noted that Aryabhata mentioned lanka here to denote at equator. As the movement of star may not be exactly towards the west closer to the poles. Also same is applicable for the stars position relative to Earth's rotation axis relatively. Refer stanza 31 of Ganita pada for relative motion as this stanza is derived from that. Aryabhata was the first during his time to share Earth's rotation and relative motion of stars due to this rotation.

Stanza 10:

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उदयास्तमयनिमित्तं नित्यं प्रवहेण वायुना क्षिप्तस् ।

लङ्कासमपश्चिमगस्थपञ्जरस्सग्रहस् भ्रमति ॥ १० ॥

"The entire structure of asterism together with the planets were moving exactly towards the west of lanka, being constantly driven by the provector wind, to cause their rising and setting"

This is continuation of earlier stanza. Here pravahavayu is the wind that impels the planets to move towards their ucca.

Stanza 11:

मेरुस्योजनमात्रस्रभाकरशिमवता परिक्षिप्तस् ।

नन्दनवनस्य मध्ये रत्नमयस्सर्वतस्वृत्तस् ॥ ११ ॥

"The Meru is exactly one yojana is height. It is light producing, surrounded by the Himavat mountains, situated in the middle of nandana forest, made of jewels and cylindrical in shape"

Probably Aryabhata is pointing North pole as mount meru here and colorful jewels are Auroras seen at North pole. It is not clearly on what is mount Meru which is mentioned in various Hindu texts. Aryabhata would have guessed that it is at North pole.

9. Interesting thoughts

Prithvi:

There is a famous thought, if you think too deep about something then you become that thing. We depend so much on substance and we follow lifecycle of a substance. Eg: If I ask you, how much amount of water was there in this Earth 1000 years before then you would wonder And say that water doesn't go out of this Earth. So, we consume but eventually it goes back to Earth and the same water purified by mother Earth comes back, without its earlier impure trace. So, amount of water is same in Earth. Another one, you know, I like Helium balloons because most of helium in balloons are the same one which existed around 3min from creation of this universe, Touch of god (ofcourse it is there is all which exists). Coming back to water, similar to water, number of Atmas in this universe is same and eternal. Water goes inside our body and comes out with impurities again goes inside another body and so on. We depend more on substance and follow the same cycle.

Stanza 12:

स्वर्मेरू स्थलमध्ये नरकस्वडवामुखं च जलमध्ये ।

अमरमरास् मन्यन्ते परस्परं अधस्थितास्त्रियतम् ॥ १२ ॥

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"The heaven and the Meru mountain are at the center of the land. The hell and the Badavamukha are at the center of water. The gods and the demons consider themselves positively and permanently below each other"

Considering heaven as North pole as most of the land in Earth is towards the north and hell as south pole which is surrounded by water. Even veda describe that heaven is towards the North.

Now interestingly one day of demigods is one human years. At North pole, one day is six months and night is six months. Hence, Aryabhata considered that Meru and heaven at Northpole and the opposite of it as hell.

Other interesting facts to note is that Mount Meru is considered as the center axis of Earth rotation as the entire cosmos revolve relatively around the axis. There is a Mount Meru in Tanzania, which roughly corresponds to the geographic Center of the Earth. The Mountain is even worshipped by the local tribes as an abode of Gods. In the country Kenya, we also find a town with the same name. Another ancient Sanskrit text, the Narpatijayacharya mentions Sumeru as being present in the middle of the Earth, but not visible to humans. Aryabhata would have taken Meru from other references and considered North pole to be the location as it matches the one year as one day equation.

Stanza 13:

उदयस्यस्लङ्कायां ससस्तमयस्सवितुरेव सिद्धपुरे ।

मध्याह्नस्यमकोट्यां रोमकविषये अर्धरात्रस् स्यात् ॥ १३ ॥

"When it is sunrise at lanka, it is sunset at siddhapura, midday at yavakoti, and midnight at romaka"

The time-distance relation is shared by Aryabhata. Place mentioned in the stanza are supposed to lie on equator and separated by one quarter of Earth's circumference on the equator.

Stanza 14:

स्थलजलमध्यात्लङ्का भूकक्ष्यायास् भवेत् चतुर्भागे ।

उज्जयिनी लङ्कायास्तद् चतुरंशे समौत्तरतस् ॥ १४ ॥

"From the centers of the land and water, at a distance of one quarter of Earth's circumference lies Lanka; and from Lanka, at a distance of one fourth thereof lies Ujjayini"

From above stance, it is clear that the prime meridian passes through Lanka and Ujjain. The latitude of Ujjain as per Aryabhata is at 22.5 deg from the Lanka towards North. However, as per Brahmagupta, it is 24deg.

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It can be noted that the location of Ujjain is important as it may define cape of cancer line. Also, Brahmagupta would have observed that the No shadow noon cannot be observed North of Ujjain. However, Aryabhata would have observed that Ujjain is slightly inside that line (cape of cancer).

Considering the refraction of sunlight it is surprising to see the selection of place for astronomical studies in such older times. They need clear night sky and a location with no mountain nearby to hide Sunlight to study their movement.

Stanza 15:

भूव्यास अर्धेन ऊनं दृश्यं देशात्समात्मगोल अर्धम् ।

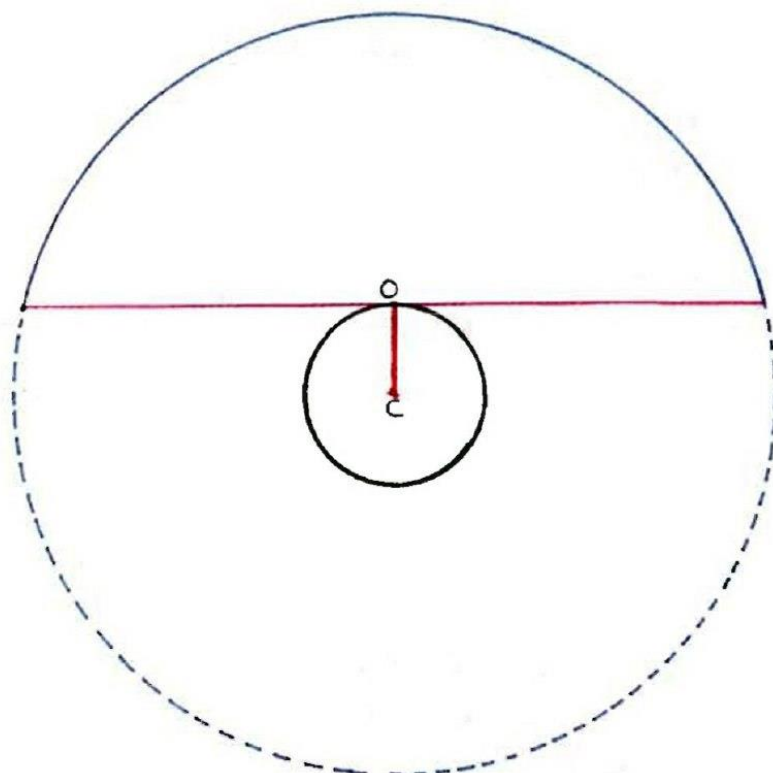
अर्धं भूमिच्छत्रं भूव्यास अर्धाधिकं च एव ॥ १५ ॥

"One half of the Bhagola as finished by the Earth's semidiameter is visible from a level place which is free from any obstruction; the other one half as increased by the Earth's semidiameter remains hidden by the Earth"

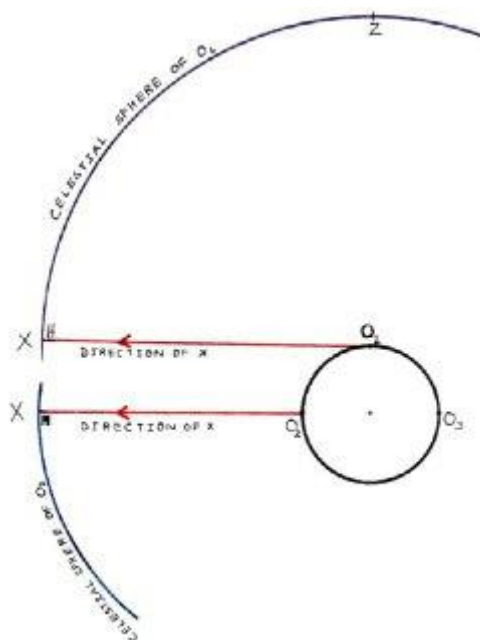
Let us start with few terms. The Sanskrit terms for the celestial sphere are bhagola, i.e., sphere of the stars [bha-star or planet; gola- sphere], and khagola, i.e., hollow sphere or sphere of the sky or celestial sphere [kha- hollow, sky, heaven]. The term bhagola was used for the celestial sphere centred at the Earth's centre, while khagola denoted the sphere centred at the observer. Aryabhata began the chapter "Gola" with a brief description of the bhagola and khagola and used them to demonstrate the motion of the celestial bodies, the bhagola was used for describing the motion of the Sun, the Moon, and the planets in their orbits; the khagola for the apparent daily motion of the heavenly bodies due to Earth's rotation. Now let us also see parallax error while observing sky. The geocentric model of celestial sphere, presenting the universe as it appears to the observer, provides a convenient framework for the computational study of those astronomical phenomena which essentially involve the directions of celestial objects when viewed from the Earth. Modern texts on basic astronomy, therefore, begin with a chapter on the celestial sphere. In standard planetarium shows, the relative positions (i.e., directions) of celestial objects are depicted on a dome representing the celestial sphere. Observers at different places on Earth get different pictures of the celestial sphere. This is because the horizontal planes at two distinct points on the Earth's surface are different. Two observers at different spots see different portions of the sky. Moreover, even if a star is seen by both observers, its orientations towards the two horizons are different. For a simultaneous analysis of the situations at different places, it would be desirable to envisage a common centre of observation. The centre of the Earth is a natural choice. Therefore, while it is undoubtedly convenient to consider the sphere centred at the observer at a place O, it might also be necessary to reduce the observed directions of celestial bodies (as observed from O) to the corresponding geocentric directions- the directions of the celestial objects that would be observed by a hypothetical observer at the centre of the Earth C positioned along CO (i.e., parallel to the observer at O). The angle between the observed direction of a heavenly body

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X and its geocentric direction, i.e., the angle subtended by CO at X, is called the geocentric parallax of X.



BHAGOLA



"One half of the bhagola, diminished by the Earth's radius, is visible from a level surface. The (view of) other half, increased by the Earth's radius, is cut off by the Earth". The focus on bhagola for the Sun, the Moon, and the planets, is perhaps a reflection of the concern for accuracy (refer geocentric parallax). Refer second image, for a person at O1 observing Sun at the horizon, observer is

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actually observing from Earth's surface but not from a view point from the center of Earth or O2.

Stanza 16:

देवास् पश्यन्ति भगोल अर्ध उदच्चेरुसंस्थितास्सव्यम् ।

अर्ध तु अपसव्यगतं दक्षिणबडवामुखे प्रेतास् ॥ १६ ॥

"The gods living in the north at the Meru mountain, at the north pole, see one half of the bhagola as revolving from left to right; the demons living in the south at the Badavamukha, at the south pole, on the other hand, see the other half as revolving from right to left"

As we have seen what is a bhagola in earlier stanza now it is important to understand the motion referred here. Bhagola revolution here denotes the movement of the stars excluding Sun. There are few assumption here as the stars are considered fixed and the revolution is due to the revolution of Earth about its axis. The relative positions of the stars appear to be fixed; i.e., the angle subtended by two such stars does not seem to change with time. Hence, the stars are referred to as fixed stars to distinguish them from other celestial objects (e.g., planets). Ancient Indian astronomers used the term nakshatra for fixed star and taragraha for a planet. In reality, the stars travel in space in different directions at different velocities. However, all stars (other than the Sun) are several light years away from Earth. Due to the enormous distances, the relative angular displacements between the stars become too minute to be noticed by the unaided human eye. Only around 200 stars are known to have an angular motion at a rate exceeding 1 second per year. But such tiny shifts in relative positions can no longer be ignored when one considers huge time intervals.

So, Aryabhata, in this stanza shared the movement of Bhagola as seen by an observer standing at North and South pole facing towards Equator. We know these things now and the content of this stanza may look simple but considering his timeline to conclude on this point he had to observe and experiment the same with simulation. Remember, we know the history of science where people got heavily criticized when they shared that Earth is not flat but a sphere. Aryabhata in his work has shared many more information further to that. We will see information on the simulation he did in upcoming stanzas.

Stanza 17:

रविवर्ष अर्ध देवास् पश्यन्ति उदितं रविं तथा प्रेतास् ।

शशिमास अर्ध पितरश्शशिगास्कुदिन अर्ध इह मनुजास् ॥ १७ ॥

"The gods see the Sun, after it has risen, for half a solar year, so is done by the demons too. The manes living on the Moon see the Sun for half a lunar month; Men here see it for half a civil day"

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All stanza in Aryabhata are either by precise observation or by logical conclusion of the observations made by him. In this stanza, first line is on the solar day time scale as seen by people in Heaven and hell. As seen earlier Heaven and hell are located at the North and South pole respectively. We know that above arctic circle they can see Sun for six month. For Aryabhata to conclude on this he would have to consider the tilt of the Earth for self rotation and revolution of Earth around Sun in different axis. Also, this is to be done without being an observer at those locations. With shadow observation this could have been made by him.

Next, Solar day on Moon. Let us see what is tidal locking, because this can be a method Aryabhata followed to conclude of solar day time scale on Moon. A lunar day is the period of time for Moon to complete one rotation on its axis with respect to the Sun. Due to tidal locking, it is also the time the Moon takes to complete one orbit around Earth and return to the same phase. Tidal locking (also called gravitational locking and on-orbit locking), in the best-known case, occurs when an orbiting astronomical body always has the same face toward the object it is orbiting. This is known as synchronous rotation: the tidally locked body takes just as long to rotate around its own axis as it does to revolve around its partner. For example, the same side of the Moon always faces the Earth, although there is some variability because the Moon's orbit is not perfectly circular. Usually, only the satellite is tidally locked to the larger body. However, if both the difference in mass between the two bodies and the distance between them are relatively small, each may be tidally locked to the other; this is the case for Pluto and Charon. Now, by observation Moon's scars, one can conclude that Moon is tidal locked. By that, Moon's relative rotation about Sun can be concluded. As Aryabhata shared, on Moon, a solar day is half a lunar month which is about two weeks. Last point is about our civil day time scale which we all observe every civil day. All minute dust in air gets accumulated at the poles. Note that dust got accumulated at the poles during earlier ice age which is also a scientifically accepted theory. So, let us say, based on our karma, our atma will either move towards North or South pole. South pole is hell surrounded by water which is colder than north pole. North pole is heaven which is warmer than south pole and much better for life. Let us say with special grace if atma can get a special power of travelling forward and backward in light with light itself as medium. Also much faster than light itself. In such cases, atma can travel backwards in Moon's light and reach Moon which can be pitruloka. If atma travels in Sun's light and passing through Sun light to the end of celestial sphere through stars then it can reach moksha. Now check Aryabhata's above stanza. You may see that time scale considered for different lokas in the path of liberation to heaven, hell or moksha may also match. It is astonishing to see the observation on Solar day time scale by Aryabhata at poles and at Moon.

Stanza 18 & 19:

पूर्वापरं अधसूर्ध्वं मण्डलं अथ दक्षिणोत्तरं च एव ।

क्षितिजं समपार्श्वस्थं भानां यत्र उदयास्तमयौ ॥ १८ ॥

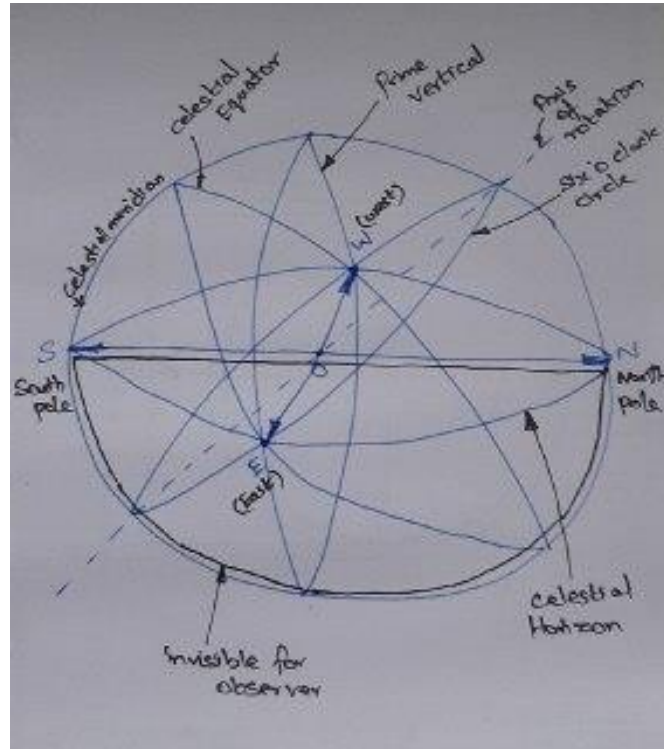
A Brief on ARYABHATIYA

पूर्वापरदिशलग्नं क्षितिजातक्षाग्रयोश्च लग्नं यत् ।

उन्मण्डलं भवेत्तत्क्षयवृद्धी यत्र दिवसनिशोस् ॥ १९ ॥

"The vertical circle passes through the east and west points is the prime vertical, and the vertical circle passing through the north and south points is the meridian. The circle which goes by the side of the above circles and on which the stars rise and set is the horizon. The circle which passes through the east and west point and meets at a distance equal to the latitude from the horizon is called equatorial horizon or six o'clock circle (उन्मण्डलं, Unmaṇḍalam) on which the increase and decrease of the day and night are measured"

Let me explain something on the celestial sphere before we go inside this stanza. Now, considering the radius of the celestial sphere, radius of Earth is very small and be considered negligible. Also, the height of the observer is negligible even when compared to the Earth radius, so let us consider the view point of observer as center of the celestial sphere. Note view direction exactly above the observer may change based on the location of the observer.



Refer drawn image below for clarity. The great circle of celestial sphere in which the horizontal tangent plane to Earth's surface, at position of the observer, meets the celestial sphere, is called the celestial horizon (क्षितिजं, Kṣitijam). The observer sees only the portion of the celestial sphere above the horizon. In image, the invisible portion of the celestial sphere and its circles are indicated by black inner line. The rising and setting of the stars and planets take place on the horizon towards the east and west respectively. The great circle of the celestial sphere whose plane is perpendicular to the axis of the celestial sphere (line joining the celestial poles) is called the celestial equator. Thus, as with the axes of rotation, the plane of the celestial equator is parallel

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to the plane of the Earth's equator (niraksa), the two planes practically coincide. Half the celestial equator is above the celestial horizon. The celestial poles are the poles of this great circle. Thus each fixed star traverses a circular orbit parallel to the celestial equator maintaining a constant angular distance from the celestial poles. The celestial equator is the fundamental great circle used for defining measures of time. The zenith is the point on the celestial sphere vertically overhead. Thus the line joining the zenith and the observer is a vertical line perpendicular to the plane of the horizon. Any great circle through the zenith is called a vertical great circle. The vertical great circle through the celestial poles is called the observer's meridian or celestial meridian. The point below the celestial north pole where the observer's meridian meets the horizon is called the North point. The point on the horizon opposite the North point is called the South point. The meridian is thus the vertical circle through the North and South points. The points of intersection of the celestial equator and the horizon are the East and West points -the East point lies to the right of the North-South line for an observer facing North; the West point lies to the left. These four points are called cardinal points. The vertical great circle through the East and West points is called the prime vertical. The great circle through the celestial poles and the East and West points is called the six o'clock circle. Aryabhata defined the prime vertical, meridian and horizon in stanza 18, the six o'clock circle in stanza 19; while the equator is implicit in stanza 1(refer earlier post). The horizon is pictured as an encircling cord around the prime vertical and the meridian. The suggestive imagery is interesting. Amidst the apparent rotation of the celestial sphere, the horizon remains firmly fixed in the middle of the sphere at right angles to the prime vertical and the meridian. Not related to our subject discussed, but it is also to be noted that word selection of Aryabhata and converting the above content into four lines in Sanskrit, considering that he placed perfectly what he wanted to convey ie his observation and definitions.

Stanza 20:

पूर्वापरदिश्रेखा अधश्च ऊर्ध्वा दक्षिणोत्तरस्था च ।

एतासां सम्पातस्द्रष्टा यस्मिन् भवेत्देशे ॥ २० ॥

"The east -west line, nadir-zenith line and the north south line intersect where the observer is"

As discussed in earlier stanza here Aryabhata keeps the observer at the center of Khagola. Nadir-zenith are the intersecting points of prime vertical with the celestial meridian.

10. Interesting thoughts

Good stress is something which excites you and is of interest to you. Good stress are beneficial to health. When does a bad stress change into good? Bad stress when approached positively can be changed to good. What kind of activities has some kind of stress? Any karma, it is upto us to convert to good or bad. If outcome of our karma define whether, it is good or bad then how to control our thoughts and make it good? Attitude towards Kainkaryam to God can solve. Any outcome is by

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God and always for our good and take this deep inside. These are some thoughts which I have on good stress.

Stanza 21:

ऊर्ध्वं अधस्ताद्भुज्यं दृश्मण्डलं ग्रहाभिमुखम् ।

दृक्षेपमण्डलं अपि प्राचलग्रं स्यात् त्रिराशिऊनम् ॥ २१ ॥

"The great circle which is vertical in relation to the observer and passes through the planet is called drnmandala. The vertical circle which passes through that point of the ecliptic which is three signs behind the rising point of the ecliptic is called drkksepavrtta"

1 sign is 30degrees. Three signs here denotes 90deg or perpendicular to the rising point of the ecliptic. Refer image shared in earlier stanza. In this stanza, it is with respect to the location of the observer on Earth. Great circle passing through the observer and vertical is drnmandala.

Stanza 22:

काष्ठमयं समवृत्तं समन्ततस्समगुरुं लघुं गोलम् ।

पारततैलजलैस्तं भ्रमयेत्स्वधिया च कालसमम् ॥ २२ ॥

"The sphere (गोलम्) which is made of wood, perfectly spherical, uniformly dense all around but light in weight, should be made to rotate keeping pace with time with the help of mercury, oil and water by the application of one's own intellect"

Aryabhata not only impressed us on Mathematical and Astronomical subject but now on the Engineering field too. Here he shared how a simulation to understand the Earth, Sun and its shadow moves as the Sun moves with time.

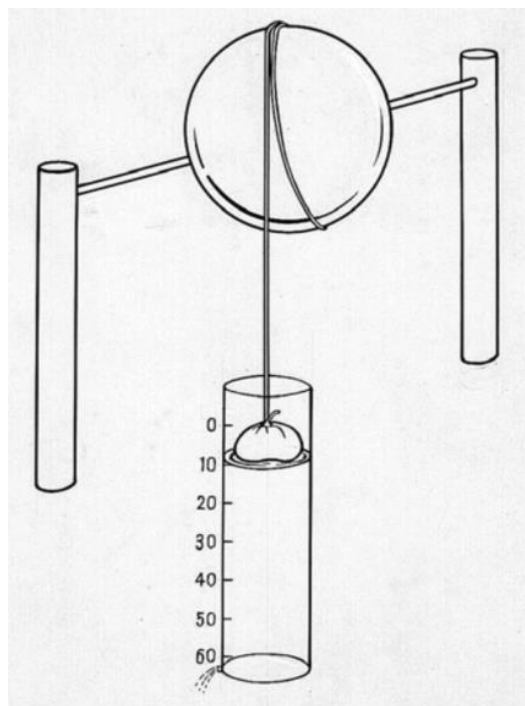
Let us put some intellect into it as shared by Suryadeva.

Let us set up two pillars on the ground. One towards the south and the other towards the north. Now mount an iron needle (rod) to form the axis of rotation. Take a wooden sphere as shared by Aryabhata and put holes for north pole and south pole. Pour some oil into North pole and south pole holes so that the sphere may rotate smoothly. Then, underneath the west point of the sphere, dig a pit and put into it a cylindrical jar with a hole at the bottom and as deep as the circumference of a sphere. Fill it with water. Then fix a nail at the west point of the sphere, tie a thread at the west point and wind it along the equator down to east and then bring back to west, then take it down again and tie it to a dry hollow gourd. Fill the gourd appropriately with mercury such that it floats on water inside the cylindrical jar placed underneath. Now the ground is set for the play. The size of hole made in the cylindrical jar should be such that the time taken for half of the water to outflow should be 12 hours and for next half another 12 hours. Looks like a difficult task then checkout Ancient Indian easter clock in internet. It may surprise you further. So, coming back, when the hole is open, water level will

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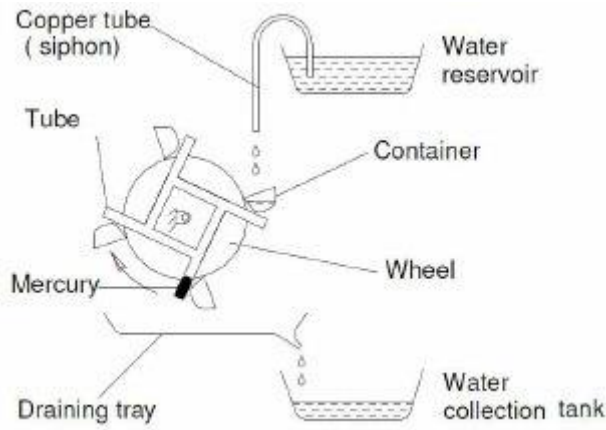
drop gradually which will make the mercury gourd to go down pulling the thread to rotate the sphere. In exactly 12 hours, sphere is rotate by half revolution and in next 12 hours it will complete one revolution. Now, globe on top of another globe is ready for you to understand solar day and night.

Surprisingly, even after sharing such experimental observation too, post Aryabhata timelines, many still believed that Earth is flat.



Indian astronomers laid special emphasis on the importance of the sphere Gola, referring to both the concept and the physical model of the celestial sphere. To quote by Lalla, "No astronomy treatise is complete without a section on the sphere of the universe. The astronomers stress that this sphere is specifically needed for mathematical computations and that those who wish to study the planets must be experts on the sphere. The mean motions of the planets and other celestial objects are clearly perceptible on this sphere, especially to the scholar who has mastered the science of their geometrical representation. How can the ignorant, who know neither mathematics nor the fundamental principles of the celestial sphere, ever detect the motion of the planets? An astronomer without the knowledge of this sphere is like a disputant without the knowledge of grammar, a sacrificer without the light of the Vedas and a physician without the experience of an active practitioner. He who acquires a comprehensive knowledge of the celestial sphere containing the Sun etc. sees, in front of his eyes as it were, the whole universe; he beholds it decorated beautifully with a wide variety of exquisite phenomena. He gets spiritually enriched and attains moksha (Liberation); he becomes famous."

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Stanza 23:

दृशगोल अर्धकपाले ज्या अर्धेन विकल्पयेत्भगोल अर्धम् ।

विषुवत्जीवाअक्षभुजा तस्यास्तु अवलम्बस्कोटिस् ॥२३॥

"Divide half of the bhagola lying in the visible half of the khagola by means of Rsine so as to form latitude triangles. The Rsine of the colatitude is the upright of the same triangle"

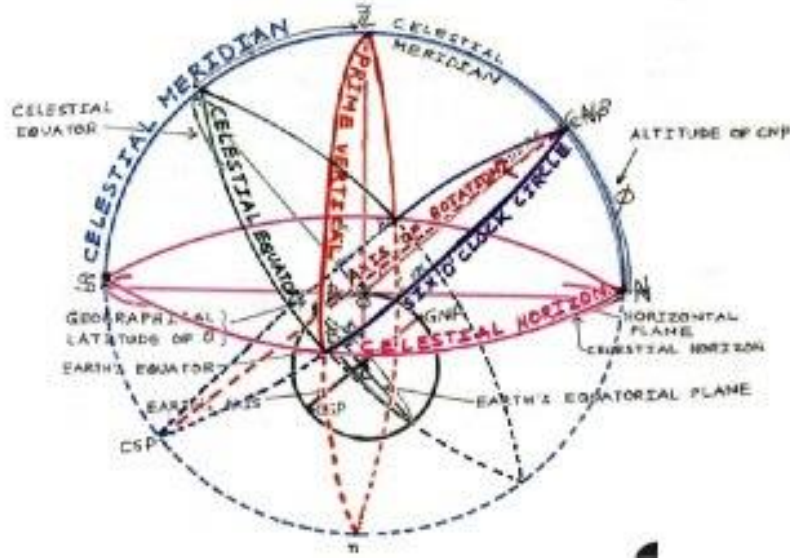
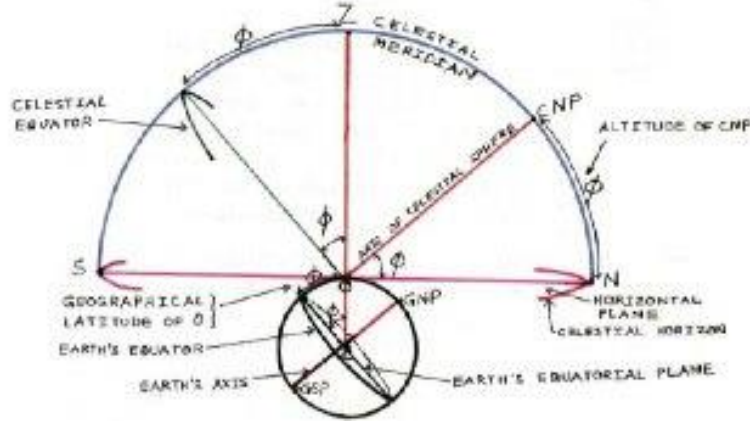
Note as shared in earlier stanza, bhagola is Sphere of Earth and khagola is sphere of asterism.

Now, this stanza is very important as our reference for any calculation depends on this. Refer image while reading below.

When a heavenly body is in the six o'clock circle, the perpendicular dropped from it on the plane of horizon is called unmandalasanku. The distance of the foot of the perpendicular from the east tallest line is called the first part of agra. The distance of the heavenly body from the rising-setting line is called the earthsine. distance of the heavenly body from the rising-setting line is called the earthsine. When

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the heavenly body is on the prime vertical, the perpendicular dropped from it on the east-west line is called samasanku. The perpendicular dropped from it on the rising-setting line is called taddhrti; the distance between the east-west line and the rising-setting line is called agra. From the foot of the samasanku on the taddhrti, the latter is divided into two parts called upper and lower; when a perpendicular is dropped from the foot of this perpendicular on the samasanku, the latter is divided into two parts called upper and lower; when from the same foot a perpendicular is dropped on the agra, the latter is divided into two parts called the 'first part of agra' and the 'other part of agra'.



Refer image superimposed on the earlier image for reference. You can how khagola and bhagola are put together.

Stanza 24:

इष्टापक्रमवर्गं व्यास अर्धकृतेस् विशोध्य यत्मूलम् ।

विषुवतुदच्छिणतस्ततहोरात्र अर्धविष्कम्भस् ॥ २४ ॥

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"Subtract the square of the Rsine of the given declination from the square of the radius and take the square root of the difference. The result is the radius of the day circle, whether the heavenly body is towards the north or towards the south of the equator"

That is,

$$\text{Day circle} = \sqrt{R^2 - (R \sin(\text{del}))^2}$$

$$R \sin(\text{del}) = R \sin(\text{lat}) \times R \sin(24) \div R$$

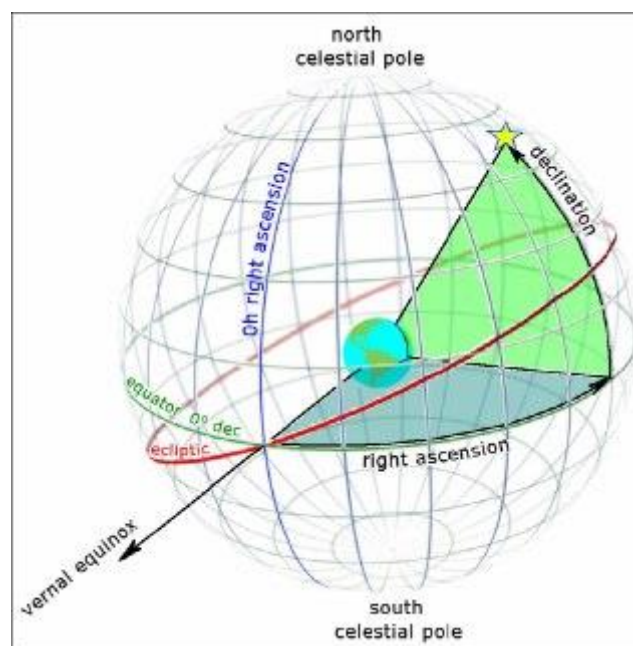
Where, del is declination of the heavenly body and lat is the Sun's tropical latitude. Second formula is based on the rule of three shared in Ganita pada. Also, 24deg can be corrected to our current tilt of Earth.

Refer link attached to see calculation of day length based different location.

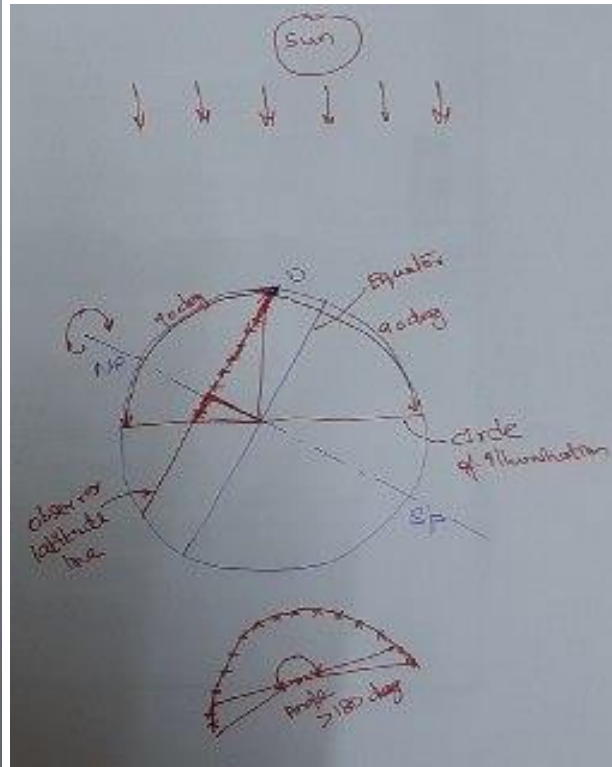
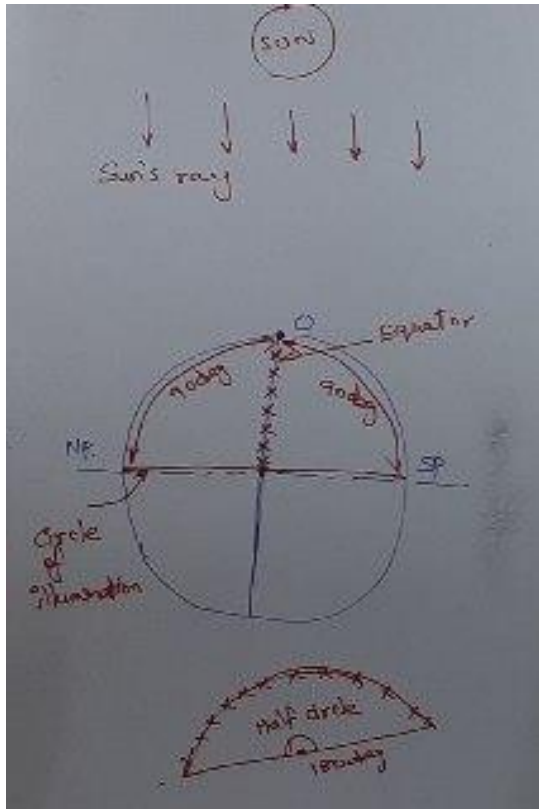
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Aryabhatiya_Thane_Chennai.xlsx

Before go into the next stanza, let us understand the day circle notation which was shared by Aryabhata. Same is even followed for day length calculation. Day and night completes one civil day which is 360deg revolution of Sun around Earth. So, 24 hrs in a day can be considered interms of Sun's angular position as 15deg per hour. For Sun to cross 15deg of longitude, roughly it takes 1 hour. This is how time is calculated based on the our location.

Like any place on Earth, every object in the sky has two numbers that fix its location called right ascension and declination, more generally referred to as the object's celestial coordinates. Declination corresponds to latitude and right ascension to longitude.



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Let's use our imaginations and picture the latitude–longitude grid on the planet as the surface of a flexible, transparent sphere. The equator, which marks the 0° latitude line, now circles the sky as the celestial equator, while the north and south celestial poles hover over either end of the planet's polar axes. Viewed from Earth's equator, the celestial equator begins at the eastern horizon, passes directly overhead and drops down to the western horizon. Since we're inside a sphere, it would continue around the backside of the Earth as well. Refer image. Why is day circle or day length required? How the day length changes based on Sun's declination and our position?

Consider that you are at the equator. Sun is without any declination and so it is exactly over equator. Now, consider that it also over your head. For any observer on Earth, entire Earth is below him and he is on top of the Earth. In this case, Sun's rays will touch exactly half of the sphere i.e. 180° longitude will be day. You can move to either side of the Earth for that 180° where the Earth will have day time. So in this case day length and night length are same 12 hrs and 12 hrs each. Refer second image.

Refer the next image, Let us consider that your position is slightly shifted from equator, let us say 10° North. Now, when the sun is on top of you, this means Sun is also declined from the equator matching your latitude position. Refer the circle of illumination cutting the bhagola at the center. However, considering the axis of rotation of Earth, checking where your latitude circle intersects the circle of illumination. Now, if you travel parallel to Equator along your latitude circle then you can travel for a longitude more than 180° and you will find that it is still day. This means that the day length along that latitude is longer. So when Sun is declined towards North and

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your latitude is also towards north then day length is more. When sun is declined towards south and your latitude is towards north then day length is shorter than 12 hrs.

Check the last image, you can see that this day length variation depends on Sun's declination, your latitude and Earth's tilt. There are other factors like refraction, radius of Sun, radius of Earth, etc but for understanding let us stick to this level. Since, our ancient astronomy has nadi and vinadi as time unit which come out to be 3600 vinnadi for day and night. Multiplying the day length angle by 10 will give the day length in vinnadi direction.

11. Interesting thoughts

Single family:

All numbers are somehow related to 0,1 and infinity.

Any number multiplied by 0 is 0.

Any number multiplied by 1 is the same number.

Any number divided by infinity is zero. Etc.,

So, all infinite numbers are part of a single family. God also possesses the property of 0, 1 and infinity. Similarly there are so many people in this world and all are related somehow to god.

Vasudhaiva kutumbakam

Stanza 25:

इष्टज्यागुणितं अहोरात्रव्यास अर्ध एव काष्ठान्त्यम् ।

स्वाहोरात्र अर्धहतं फलं अजालङ्काउदयप्राच्यास् ॥ २५ ॥

"Multiply the day radius corresponding to the greatest declination on the ecliptic by the desired Rsine of one, two and three signs and divide by the corresponding day radius. The result is the Rsine of the right ascension of one, two and three signs measured from the first point of Aries (Mesa) along the equator"

Earlier stanza, we saw diurnal circle of the heavenly body. Also, we saw with example of Sun, day circle and the link between day circle & day length. Now in this stanza the position of the signs in terms of right ascension is shared here by Aryabhata.

Let A, B and C denote the right ascensions of one sign, two signs and three signs, respectively, and D1, D2 and D3, the declinations at the last points of the signs Aries(Mesa), Taurus(Vrsabha) and Gemini(Mithuna), respectively.

Then,

$$R\sin A = R\sin 30^\circ \times R\cos 24^\circ \div R\cos D1$$

$$R\sin B = R\sin 60^\circ \times R\cos 24^\circ \div R\cos D2$$

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$$R \sin C = R \sin 90^\circ \times R \cos 24^\circ \div R \cos D_3$$

Above formula is valid for an heavenly body on the celestial sphere. Since the sign are placed equally spaced around 360deg, we divided uniformly. Form a triangle with declination, horizon and a sign in khagola shared earlier to get the formula above.

Take refer from Ganita pada for triangle and Rsine relation.

Now,

$$R \cos D_1 = 3366', R \cos D_2 = 3218' \text{ and } R \cos D_3 = 3141'$$

(refer Gitika pada for corresponding Rsin values at 90-angle). Hence substituting these values and simplifying,

we get

$$A = 1670', B = 3465' \text{ and } C = 5400'.$$

Consequently,

Right ascension of Aries = 1670 respirations

Right ascension of Taurus = 1795 respirations

Right ascension of Gemini = 1935 respirations.

The right ascensions of Aries, Taurus and Gemini in the reverse order are the right ascensions of Cancer(Karka), Leo(Simha) and Virgo(Kanya); and the right ascensions of the first six signs, Aries etc. in the reverse order are the right ascensions of the last six signs, Libra (Tula), etc.

Right ascension of Aries, virgo, pisces (Mina) & Libra = 1670 respirations

Right ascension of Taurus, Leo, Aquarius (Kumbha) & Scorpio (Vrscika) = 1795 respirations

Right ascension of Gemini, Cancer, capricorn (makara) & sagittarius (Dhanusa) = 1935 respirations.

Stanza 26:

इष्टाप्रक्रमगुणितां अक्षज्यां लम्बकेन हत्वा या ।

स्वाहोरात्रे क्षितिजा क्षयवृद्धिज्या दिननिशोस्सा ॥ २६ ॥

"The Rsine of latitude multiplied by the Rsine of the given declination and divided by the Rsine of colatitude gives the earthsine, lying in the plane of the day circle. This is also equal to the Rsine of half the excess or defect of the day or night in the plane of the day circle."

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We have seen on the day circle and relation between day circle and day length in stanza 24. Now, here Aryabhata shared the formula for excess or defect which is increase or decrease in day length in terms of day circle. That is,

$$\text{Earthsine} = R \sin(D) \times R \sin(\text{lat}) / R \sin(90 - \text{lat})$$

$$= R \sin(D) \times R \sin(\text{lat}) / R \cos(\text{lat})$$

$$= R \sin(D) \times R \tan(\text{lat})$$

Where D is declination and lat is latitude.

You can see how Aryabhata takes all Sine values in terms of Rsine and he has already shared the values of Rsine and method of calculating Rsine (refer Githika pada) which can directly used here for calculation.

By the 'excess or defect of the day or night' is meant the amount by which the day or night at the local place is greater or less than 30 ghatas (or 12 hours). The earthsine is the Rsine of half the excess or defect of the day or night in the plane of the day circle. Since the time is measured on the equator, one should first find the corresponding Rsine in the plane of the equator and then reduce that to the arc of the equator. The Rsine of half the excess or defect of the day or night in the plane of the equator is called carardhajya and is obtained by the following formula,

$$\text{Carardhajya} = \text{Earthsine} \times R / \text{radius of day circle}$$

The corresponding arc of the equator is called carardha and gives the amount by which the semi-duration of the day or night at the place is greater or less than 15 ghatas. Carardha is also equal to the difference between the oblique and the right ascension and so it is called as the ascensional difference. The oblique ascension is the time of rising of an arc of the equator at the local place and the right ascension is the time of rising of an arc of the ecliptic at the equator.

Carardhajya can also be expressed in terms of shadow of Gnomon as,

$$\text{Carardhajya} = 3438 \times \tan(D) \times (e/g)$$

Where, e is the equinoctial noon shadow and g is the gnomonic height. 3438 is retained as this is used by Aryabhata for his Rsine values.

Stanza 27:

उदयति हि चक्रपादस्वरदलहीनेन दिवसपादेन ।

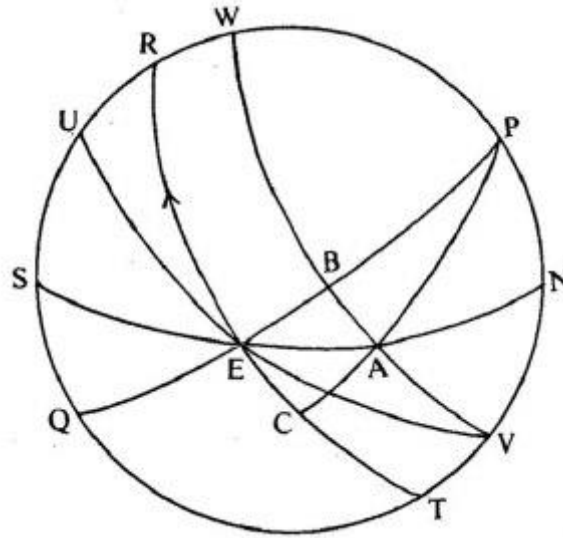
प्रथमसन्त्यस्व अथ अन्यौ तदसहितेन क्रमौल्लमशस् ॥२७॥

"The first as well as the last quadrant of the ecliptic rises above the local horizon in one quarter of a sidereal day diminished by the ghatas of the ascensional difference. The other two viz. the second and third quadrants rise in one quarter of a sidereal day

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as increased by the same i.e. the ghaṭis of the ascensional difference. The times of rising of the individual signs (Mesa, Vrsabha and Mithuna) in the first quadrant are obtained by subtracting their ascensional differences from their right ascensions in the serial order; in the second quadrant by adding the ascensional differences of the same signs to the corresponding right ascensions in the reverse order. The times of risings of the six signs in the first and second quadrants (Mesa, etc.) taken in the reverse order give the risings of the six signs in the third and fourth quadrants (Tula, etc.)"

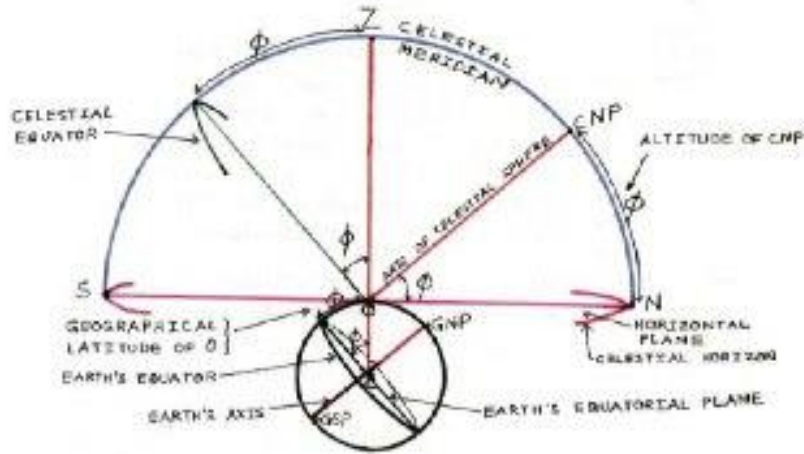
In Aryabhatiya the content builds up stanza by stanza and so each stanza is important to understand the next. Refer the first figure of Khagola now if we introduce the day circle in that then it will look like figure two.



Let figure 2 represent the Khagola for the local place. (Refer first figure for reference) SEN is the horizon, RET the equator, UEV the ecliptic and PEQ the equatorial horizon. The small circle WBV is the day circle through V (the end of the first quadrant of the ecliptic). EV is the first quadrant of the ecliptic. At the moment the first point of Mesa (A) coincides with E. With the motion of the Celestial Sphere E will move along the equator and V along the diurnal circle (day circle) in the direction of the arrowhead. When V reaches A, the whole of the first quadrant of the ecliptic, which is at the moment on the point of rising above the horizon, will be above the local horizon. So the time of rising of the first quadrant of the ecliptic at the local place is the time taken by V in moving from V to A, or, what is the same thing, the time taken by T in moving from T to C. This time is given by the arc TC or ET-EC of the equator (because time is measured on the equator).

Refer earlier stanza for understanding Khagola (Celestial sphere) clearly. Later as we go on, we will also see star map shared in Rashtriya Panchang which is nothing but khagola with stars places in it. You can see poles, celestial equator and ecliptic in Rashtriya panchang also.

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You may wonder why to do all these calculation. But you should also question whether the birthday of yours EVERY YEAR what you celebrate is the actual the correct date after a solar year? Now then you will get to know the errors in year calculation, month and day calculation. Now all these errors in modern calendar is by taking reference of stars and signs. It may look that there are many calculation but actual our panchang shows directly the day by taking reference of stars and planets. So celebrating your date of birth or marriage anniversary by star reference is the correct way and same is followed in our Bharat.

We saw the time taken for point V to move the horizon which is nothing but time to rise point. Since ET is one-fourth of the equator, it corresponds to one-quarter of a sidereal day. So,

the time of rising of the first quadrant of the ecliptic is (one quarter of a sidereal day - ghatas corresponding to arc EC) which is same as (15 ghatas - ghatas of ascensional difference).

This ascensional difference can be experimentally verified if we know the direction of North properly. Let us see how to check it. It can be noted that how Aryabhata was an expert in geometric calculation of a sphere and experimental verification of the same by observation and a simulation (refer earlier stanza on simulation).

The first quadrant of the ecliptic is, at the point of rising above the equatorial horizon QEP. When the point V reaches the point B, the first quadrant of the ecliptic will be completely above the equatorial horizon which can be observed by the observer. The time taken by V to reach B is equal to the time taken by T in reaching E. So the time of rising of the first quadrant of the ecliptic above the equatorial horizon is equal to the arc ET of the equator, which has been just shown to correspond to one quarter of a sidereal day or 15 ghatas. This differs from the earlier conclusion by the time given by the arc EC of the equator. EC therefore gives the difference between the times of rising of the first quadrant at the local and equatorial places. EC is, hence, called the 'ascensional difference of the first quadrant' (or the ascensional difference of the last point of the first quadrant).

Hence , from earlier equation, we have,

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Time of rising of the first quadrant at the local place = 15 ghatīs - ghatīs of the ascensional difference.

When the first point of Mēṣa is at E, the first point of Tula is at the west point W. The first point of Tula will reach the point E exactly after 30 ghatīs and then the second quadrant of the ecliptic will be completely above the local horizon. Hence we have,

Time of rising of the second quadrant of the ecliptic at the local place = 30 thatīs – (15 ghatīs - ghatīs of asc. diff.) = 15 ghatīs + ghatīs of asc. diff.

Similarly, we can get,

Time of rising of the third quadrant of the ecliptic at the focal place = 15 ghatīs + ghatīs of asc. diff.

Time of rising of the fourth quadrant of the ecliptic at the local place = 15 ghatīs - ghatīs of asc. diff.

Similarly,

Time of rising of the sign Mēṣa at the local place = Time of rising of the sign Mēṣa at the equator - asc. diff. of the last point of Mēṣa.

Time of rising of the signs Mēṣa and Vṛṣabha at the local place = Time of rising of the signs Mēṣa and Vṛṣabha at the equator asc. diff. of the last point of Vṛṣabha.

Time of rising of the signs Mēṣa, Vṛṣabha and Mithuna at the local place = Time of rising of the signs Mēṣa, Vṛṣabha and Mithuna at the equator - asc. diff. of the last point of Mithuna.

From the above, we get,

Time of rising of the sign Mēṣa at the local place = Time of rising of the sign Mēṣa at the equator - asc. diff. of Mēṣa.

Time of rising of the sign Vṛṣabha at the local place = Time of rising of the sign Vṛṣabha at the equator - (asc. diff. of the last point of Vṛṣabha - asc. diff. of the last point of Mēṣa)

= Time of rising of the sign Vṛṣabha at the equator - asc. diff. of Vṛṣabha.

Time of rising of the sign Mithuna at the local place = Time of rising of the sign Mithuna at the equator- (asc. diff. of the last point of Mithuna - asc. diff. of the last point of Vṛṣabha)

= Time of rising of the sign Mithuna at the equator - asc. diff. of Mithuna.

Let A, B, C be the times of rising of the signs Mēṣa, Vṛṣabha, and Mithuna at the equator and a, b, c the ascensional differences of the same signs in their respective order. Then the times of rising of the signs at the local place are as shown below.

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Time of rising are,

Mesa & Mina = A-a

Vrsabha & Kumbha = B-b

Mithuna & Makara = C-c

Karka & Dhanusa = A+a

Simha & Vrschika = B+b

Kanya & Tula = C+c.

12. Interesting thoughts

Infinity:

$$10x^\infty = \infty$$

$$1x^\infty = \infty$$

$$0.1x^\infty = \infty$$

$$0.00001x^\infty = \infty$$

$$0x^\infty = \text{????}$$

If each particle in this universe is expanding by disintegration then the particle size will follow above steps and finally end up with that question. Find it mathematically!!!

Stanza 28:

स्वाहोरात्रैष्टय्या क्षितिजातवलम्बकऽहतां कृत्वा ।

विष्कम्भ अर्धविभक्ते दिनस्य गतशेषयोऽष्टाङ्कुस् ॥ २८ ॥

"Find the Rsine of the arc of the day circle from the horizon (up to the point occupied by the heavenly body) at the given time; multiply that by the Rsine of the colatitude and divide by the radius; the result is the Rsine of the altitude of the heavenly body at the given time elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon."

Istahrta is the distance of the heavenly body from the rising - setting line. It is the Rsine of the arc of the day circle from the horizon up to the point occupied by a heavenly body. So,

$$R\sin(\text{Sun's altitude}) = \text{istahrta} \times R\sin(90-\text{lat})/R$$

$$= \text{istahrta} \times R\cos(\text{lat})/R$$

With the help of the Sun's declination and the local latitude calculate the Sun's ascensional difference. Subtract the Sun's ascensional difference from or add that to the given time reduced to asus(1 ghati= 360asus), according as the Sun is in the northern or southern hemisphere. By the Rsine of that difference or sum multiply the day radius and divide by the radius. If the Sun is in the northern hemisphere, add the

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earth'sine to the result obtained; if the Sun is in the southern hemisphere, subtract the earth'sine from the result obtained; the result is the istahrti. Multiply that by the Rsine of the colatitude and divide by the radius; the result is the Rsine of the Sun's altitude. When the Sun is in the northern hemisphere and the given time reduced to asus is less than the Sun's ascensional difference reduced to minutes of arc, we should do as follows:

Subtract the asus of the given time from the minutes of the Sun's ascensional difference; multiply the difference by the day radius and divide by the radius. Subtract whatever is obtained from the earth'sine; the result is the istahrti. Multiply that by the Rsine of colatitude and divide by the radius; the result is the Rsine of the Sun's altitude as before.

Stanza 29:

विषुवत्जीवागुणितस्वैष्टशङ्कुस्वलम्बकेन हतस् ।

अस्तमयौदयसूत्रादक्षिणतस्सूर्यशङ्कुअग्रम् ॥ २९ ॥

"Multiply the Rsine of the Sun's altitude for the given time by the Rsine of the latitude and divide by the Rsine of colatitude; the result is the Sun's sankvarga, which is always to the south of Sun's rising-setting line"

Sankvarga is also called as Sankutala which is the base of the Sanku. It denotes the distance of the base of the Sanku from the rising-setting line.

Sun's sankvarga = $R \sin(\text{Sun's altitude}) \times R \tan(\text{latitude})$

Stanza 30:

परमापक्रमजीवां इष्टज्या अर्धऽहतां ततस् विभजेत् ।

ज्या लम्बकेन लब्धा अर्काग्रा पूर्वापरे क्षितिजे ॥ ३० ॥

"Multiply the Rsine of the Sun's tropical longitude for the given time by the Rsine of Sun's greatest declination and then divide by the colatitude; the resulting Rsine is the Sun's agra on the eastern and western horizon"

Sun's agra is the distance of rising or setting point of Sun from the East-west line.

Sun's agra = $R \sin(\text{Tropical long}) \times R \sin(24) / R \sin(90 - \text{lat})$

= $R \sin(\text{Tropical long}) \times R \sin(24) / R \cos(\text{lat})$

Stanza 31:

सा विषुवत्ज्याऊना चेद्विषुवतुदल्लम्बकेन सङ्गुणिता ।

विषुवत्ज्याया विभक्ता लम्बस्पूर्वापरे शङ्कुस् ॥ ३१ ॥

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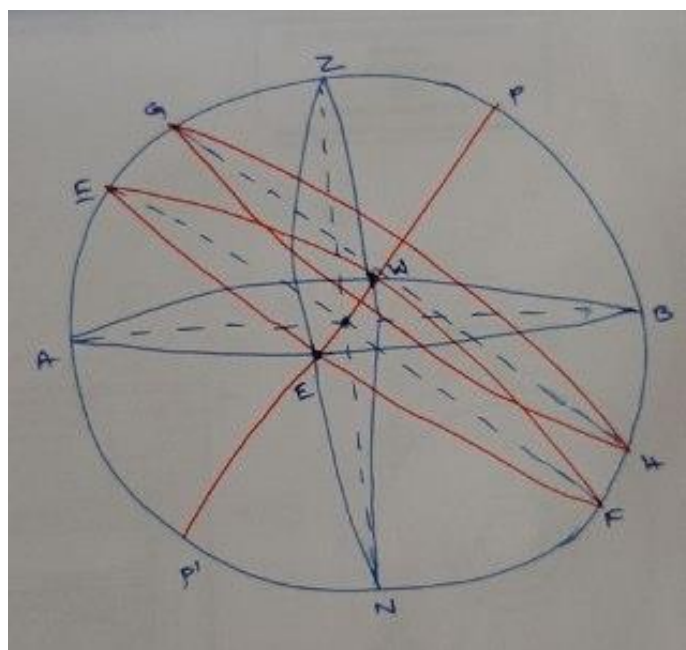
"When that agra is less than the Rsine of the latitude and the Sun is in the northern hemisphere, multiply that by the Rsine of colatitude and divide by the Rsine of latitude; the result is the Rsine of the Sun's altitude when the Sun is on the prime vertical"

That is,

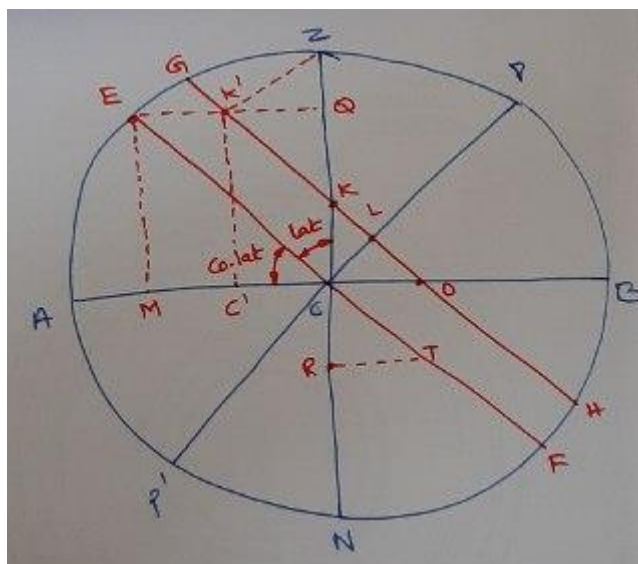
$$R \sin(\text{Sun's prime vertical altitude}) = \text{Sun's agra} \times R \cos(\text{lat}) / R \sin(\text{lat})$$

Before we go into shadows and eclipses, I will summarise diurnal motion with geometric representation and trigonometry. I will share previous few stanza's summary tomorrow. They are important to understand shadows, gnomon and eclipses which we will see later.

Before going into shadows and eclipses, let me summarise the diurnal motion and its trigonometry on sphere. In khagola, we have seen the celestial horizon, prime vertical, north-south line, east-west line and celestial equator. Refer image. For any heavenly body, we saw its motion in khagola and diurnal circle. Let us make a geometry based on the formula shared to understand it in a easy way.



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Let us draw a circle to represent khagola with center C and radius R. Z is the zenith of khagola. AB is the celestial horizon above which the visible portion of khagola is present. ACB is the diameter of the celestial horizon. EF is the celestial equator. Diameter of the celestial equator is ECF. Here all are referred as diameter as they are 2D representation of a circle. Refer third image for khagola sphere and the same is represented in 2D in second image. PP' is the poles of the celestial equator. Also, PCP' is the six o'clock circle. Let us also draw a diurnal circle GH with GH as diameter. Let us consider Sun as the heavenly body which is travelling on the diurnal circle in khagola. ZCN is the prime vertical and diameter of the prime vertical is ZN. As shown in the figure for an observer at C having celestial horizon as AB, angle ECZ is the latitude and ECA is the colatitude. Refer fourth image where bhagola and khagola are superimposed. Assume bhagola as negligible radius when it comes to khagola geometry. Now,

$$CM = R \sin(\text{lat})$$

$$EM = R \sin(\text{co-lat})$$

$$CE = R$$

Also, in figure, CL is the distance between celestial equator and diurnal circle.

$$CL = R \sin(\text{Sun's declination})$$

Point L here is the intersection point of diurnal circle and six o'clock circle. LO is the line joining that point and the horizon point on diurnal circle. Hence, LO is Earth's sine or kujya. CO which lies on the horizon is the distance between east-west line and the rising-setting line. This is called $R \sin(\text{Sun's amplitude or agrajya})$.

When Sun is at K, KC is then $R \sin(\text{prime vertical altitude or samasanku})$. KO is prime vertical hypotenuse or taddhrti.

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When Sun is at K', K'C' is $R \sin(\text{altitude})$ at any time or Sanku. C'O is the base which is called as sankutala and K'O is the hypotenuse at that time which is istahrta. Now, let Sun be at E which equinoctial point. If we have a gnomon of length CR then the equinoctial midday shadow length will be RT or palabha. CT will be it hypotenuse or palakarna. Note that there are six similar triangles in the figure. Now with this understanding we can see about shadows.

We all know that the shadow of gnomon varies in different time of the year. But you that we can find the latitude of your location with shadow? Do you know that you can find direction North, South, East and West by shadow? Do you know that you can find Earth's axis tilt by shadow?

We will see those in this post. This was used and shared by Aryabhata too. These concepts are essential for anyone to know to identify the actual solar time for doing Nityakarma. Earlier days even these are essential to do any karma as all harvesting, construction, studies, marriage or any other activities were planned properly to get better results as Sun light plays a major role in natural occurrences. First for your latitude, find the equinoctial midday shadow of a cylindrical gnomon. Find hypotenuse of the triangle formed by gnomon and shadow. Now in this triangle,

$\sin(\text{lat}) = \text{shadow length/hypotenuse}.$

For Earth's tilt, check the shadow of gnomon around the year and mark the midday shadow on every day. Mark the maximum shadow points. If you are within cape of cancer and cape of capricorn then you will get shadow on both sides. If you are out of the this then you will get in one side (mark max and min shadow in this case). Tie two threads to the top tip of the gnomon and join the thread the two shadow points. Angle between the thread will be twice that of the Earth's tilt.

For direction, draw a circle with radius 2R and place a gnomon of height R. Mark three shadows. One is the shortest of the day, shadow when the tip of the shadow enters the circle and shadow when the tip of the shadow leaves the circle. Mark the entering and leaving point. Take a thread of length equal to the distance between these above two points. Fold it to half and draw circles with half thread length as radius and entering and leaving points as centers. Mark the intersecting or touching point of these two circles. Draw line from bottom of gnomon and the intersecting point. This direction is south if the Sun is in northern hemisphere. Draw perpendiculars to this line for East and west direction.

With just the Shadows, Geometry, Sphere, and precise measurement, ancient Indian astronomers shared so many data on heavenly bodies. These are such interesting practical knowledge which we want but miss in our education system. Tomorrow, we will go to the next stanza in Gola pada.

Stanza 32:

क्षितिजातुन्नतभागानां या ज्या सा परस् भवेत्शङ्कुस् ।

मध्याह्नतभागज्या छाया शङ्कोस्तु तस्य एव ॥ ३२ ॥

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"The Rsine of the degrees of the Sun's altitude above the horizon at the midday when the Sun is in the meridian is the greatest gnomon on that day. The Rsine of the Sun's Zenith distance at that time is the shadow of the same gnomon"

This stanza forms basis for defining greatest gnomon and also used in designing a modern gnomon. Formula changes according to the position Sun in northern or southern hemisphere. Greatest gnomon is also called as maha-sanku.

The sum of the declination of the Sun and the colatitude of the observer is the meridian altitude or unnatamsa of Sun when it is in the northern hemisphere and difference of them is for southern hemisphere. Meridian zenith distance is when the altitude is subtracted from 90. Refer earlier post image for reference. Same formula can be applied when Sun is at equinox and check how the formula changes then check my earlier post on finding latitude of your location.

Gnomon was used in india long time back and there are many ancient works explaining them. You can refer Kautilya's arthashastra, section on "Measurement of space and time" explaining how to find time based on gnomon. Aryabhatiya also shares the concept in short formula. There are many other works. Sun dial and instrument to measure time are very ancient and discovered at many location in our Bharat.

Stanza 33:

मध्यज्याउदयजीवासंवर्गे व्यासदलहते यत् स्यात् ।

तद्वध्यज्याकृत्योस्विशेषमूलं स्वदृक्षेपस् । ३३ ।।

"Divide the product of the madhyajya and the udayajya by the radius. The square root of the differences between the squares of that result and the madhyajya is the Sun's and Moon's drkksepa"

This is an approximation done by Aryabhata to drkksepjya of Sun and Moon. Zenith point is taken reference in ancient astronomy as it is easy to defined this imaginary axis between Zenith and Nadir with the plum line or line of action of gravity. Experimentally, to find angle between the Zenith and any celestial object is very easy. Take a straight bamboo stick and make a thru hole at the center. Tie a plumb at the one end of the bamboo. Now see any celestial object thru the bamboo hole. Angle between the bamboo stick and the plumb line is the angle of celestial body with Zenith. This technique is one of method of nalaka yantra. This measurement show pictorial in figure 1. Same is shown in celestial sphere in figure 2.

Coming back to the stanza, Madhyajya is the Rsine of the zenith distance of the meridian ecliptic point. Udayajya is the Rsine of the amplitude of the rising point of the ecliptic. Drkksepajya is the Rsine of that point of the ecliptic which is shortest distance from zenith.

$$\text{Rsin(arc between zenith shortest distance point to meridian)} = X = \frac{\text{madhyajya} \times \text{udayajya}}{R}$$

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$$\text{Drkksepajya} = \sqrt{(\text{Madhyajya}^2 - X^2)}$$

This is valid if the Rsine of the sides formed by meridian point, zenith point and the shortest distance point of the ecliptic is right angles triangle. However, it is not so and formula is thus an approximation and inaccurate.

Stanza 34:

दृष्टक्षेपकृतिविशेषितस्य मूलं स्वदृशगतिस्कुवशात् ।

क्षितिजे स्वा दृच्छाया भूव्यास अर्धं नभस्मध्यात् ॥ ३४ ॥

"The square root of the difference between the squares of the Rsine of the zenith distance of the Sun and the Moon and the drkksepajya is the Sun and Moon's drggatijya. On account of the sphericity of the Earth, parallax increases from zero at the zenith to the maximum value equal to the Earth's semi diameter as measured in the spheres of the Sun and Moon at the horizon"

Drggatijya is the Rsine of the arcual distance of the zenith from the secondary to the ecliptic passing through the heavenly body.

$$\text{Drggatijya} = \sqrt{(\text{Rsin}(\text{Zenith distance}))^2 - (\text{drkksepajya})^2}$$

$$\text{Drkchaya or parallax in longitude in yojanas} = \text{Earth's semi diameter} \times \text{drggatijya} / R$$

$$\text{Drkchaya or parallax in latitude in yojanas} = \text{Earth's semi diameter} \times \text{drkksepajya} / R.$$

Refer image 1 and image 2. Closely observe the reorientation between the first and second image. First image shows the Earth we normally see in map with equator placed horizontally. Second image is take from the first by considering the observer and the zenith in the vertical direction with portion of the ecliptic above horizon is shown(as in khagola). Let U be the rising point of the ecliptic (udaya-lagna), T be the nonagesimal (tribhona-lagna) and M be the meridian-ecliptic point (madhya-lagna). T is at a distance of one quadrant from U along the ecliptic. The complement of the zenith distance of T, that is the arc TK, will be the required angle between the ecliptic UTM and the horizon NUEKS. This is called drkksepajya or the sine of the zenith distance of the nonagesimal. It is important to understand these notation to understand what is shared in drkksepajya, drggatijya and drkchaya.

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Angles at T and K are right angles, and it is easily seen that

$$\text{arc SE} = \text{arc KU} = \text{arc TU} = \text{arc } 90^\circ$$

so that,

$$\text{arc EU} = \text{arc SK} = \text{angle MZT} = v$$

If t and m denote the zenith distance of T and M respectively, then the required angle

$$\text{Angle TUK} = \text{arc TK} = 90 - t$$

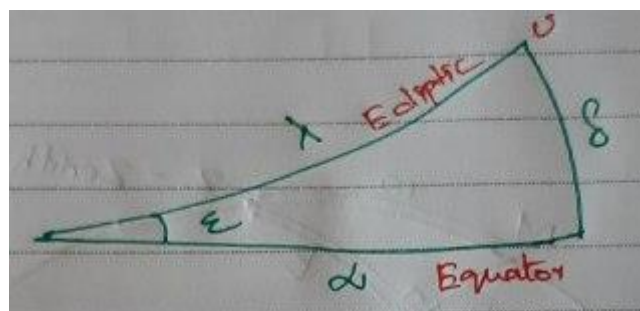
From the spherical right-angled triangle ZMT we have

$$\sin s \times \sin 90 = \sin m \times \sin v$$

$$\cos m \times \cos 0 = \cos s \times \cos t$$

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$$\cos m \times \cos 0 = \cos s \times \cos t$$

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where denotes the arc MT.

Converting all cosines to sines in above and solving above two equation for sin t, we get

$$\sin t = \sqrt{\frac{(\sin(m))^2 - (\sin(s))^2}{1 - (\sin(s))^2}}$$

Or

$$t = \cos^{-1}(\cos(m) / \cos(s))$$

From celestial sphere and observation, one can find t from either of the above equation.

According to Aryabhata, he shared svadrkksepajya which is

$$R\sin(t) = \text{svadrkksepajya} \times R / R\cos(s)$$

$$\text{And svadrkksepajya} = \sqrt{(R\sin(m))^2 - (R\sin(s))^2}$$

So,

$$R\sin(t) = \sqrt{(R\sin(m))^2 - (R\sin(s))^2} \times R / R\cos(s)$$

We know from stanza 25 of Golapada that to find the Right ascension from a given longitude and obliquity. Refer third image which is extracted from the first image. Lambda is ecliptic longitude; alpha is right ascension; eeta is obliquity of the ecliptic; delta is declination.

Now,

$$R\sin(\text{right ascension}) = R\sin(\text{ecliptic longitude}) \times R\sin(\text{obliquity of the ecliptic}) / R\cos(\text{declination})$$

Refer stanza 25 of Gola pada where Aryabhata has shared for signs. Above formula is similar to one which we saw for signs.

Comparing the similar triangle from image 3 and image 2. We can get

$$R\sin(t) = R\sin(m) \times R\cos(v) / R\cos(s)$$

Which also gives the same formula for Rsin(t) as we saw earlier. Aryabhata wanted to share for Sun considering the ecliptic of the Sun.

Stanza 35:

विक्षेपगुणाक्षज्या लम्बकभक्ता भवेत्तुणं उदक्ष्ये ।

उदये धनं अस्तमये दक्षिणगे धनं ऋणं चन्द्रे ॥ ३५ ॥

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"Multiply the Rsine of the latitude of the local place by Moon's latitude and divide resulting product by the Rsine of the colatitude: the result is the aksadrkkarma for Moon. When Moon is to the north of the ecliptic, it should be subtracted from Moon's longitude in the case of the rising of Moon and added to Moon's longitude in the case of the setting of Moon; when Moon is to the south of the ecliptic, it should be added to Moon's longitude in the case of the rising of Moon and subtracted from Moon's longitude in the the case of the setting of Moon"

When the celestial latitude of heavenly body is small then Aksadrkkarma is considered based in the formula shared in stanza.

Aksadrkkarma is the arc length of the ecliptic between the point of intersection of horizon-ecliptic and hourcircle-ecliptic.

Aksadrkkarma = $\frac{\text{Rsin}(\text{lat}) \times \text{Moon's latitude}}{\text{Rcos}(\text{lat})}$
= $\text{Rtan}(\text{lat}) \times \text{Moon's latitude}$.

13. Interesting thoughts

What is a clay pot? We can define a pot based on shape, purpose and raw material. Clay pot of a particular raw material and size will have mass(kg). But, what it actually made of & how does it get mass? Pot is made of clay and mass depends on the amount of clay. What is clay? How clay gets mass? Clay is made of silicate rocks and other sand. These are made of chemical earthly elements (Earth) with liquid (Water) mixed to make shape. Particles of clay are of uneven shape so it also has Air gap (Air) in it. How do clay get mass and what chemical elements are made of? All chemical elements are made by fusion of hydrogen and gets mass based on the level of fusion and number of hydrogens involved in fusion. How do hydrogen get its mass and what hydrogen is made of? Hydrogen is made of proton, electron and neutron placed away with certain space(Space/ether). So Hydrogen gets its mass by these elements. How do these elements get their mass? We can keep going like this, we will end with Paramatma which one can only and has to believe. Like the way electrons revolve neutrons, even planets revolve the Sun. All these elements, object, planets obey rules set by Paramatma, who is not bound by any rules. Mass, gravity, etc., are terms used to share our understanding on these rules which we are not clear yet. Space, fire, air, water, earth are five elements of any object which depend on each other and created one after/by another. All these elements are bound by Paramatma and his rules. There is one more thing which is missing here. That is life inside living being. Life is also an unknown thing to us as the source of life is still a mystery. We are bound by the rules set by Paramatma for these objects as we believe so. So, by our wrong belief, objects bound us and our life is controlled by these objects. We are different from other object. Did we ever question that are we bound by the rules set for objects? What are rules set for us? We are supposed to serve Paramatma. Our duty and rules set for Jeevatma are to serve the Paramatma. One who understands this, is not bound by the control of objects. He serves the Paramatma as intended in Vaikunta.

Om Namo Narayanaya

A Brief on ARYABHATIYA

Stanza 36:

विक्षेपापक्रमगुणं उत्क्रमणं विस्तर अर्धकृतिभक्तम् ।

उदचृणधनं उदचयने दक्षिणगे धनं ऋणं याम्ये ॥ ३६ ॥

"Multiply the Rversed sine of the Moon's tropical longitude by the Moon's latitude and also by the Rsine of Sun's greatest declination and divide the resulting product by the square of the radius. When the Moon's latitude is North, it should be subtracted from or added to the Moon's longitude, according at the Moon's ayana is north or south; when the Moon's latitude is south, it should be added or subtracted respectively"

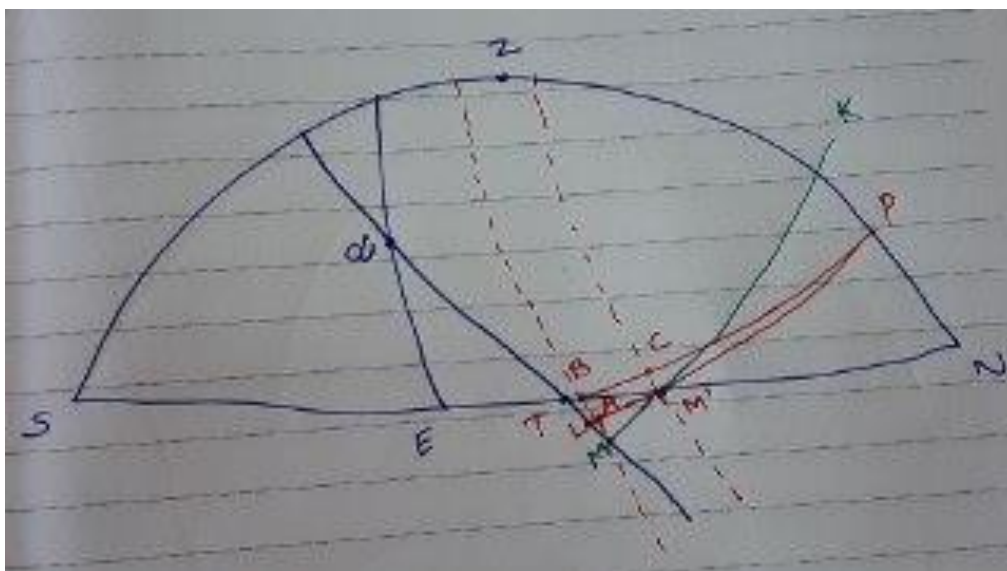
That is,

$$\text{Ayanadrkkarma} = \text{Rvers}(90 + \text{Moon's tropical longitude}) \times \text{Moon's latitude} \times \text{Rsin}(24) / (\text{R}^2)$$

24deg is considered as Sun's greatest declination as shared by Aryabhata which was valid and closer during his time.

We will see the geometric representation of above two stanza in next post. After that we will see about eclipses and shadow length in upcoming stanzas.

Khagola is shown in first figure. Angle made by the pole to celestial horizon is local latitude and altitude is the angle made by the celestial equator with the horizon. Line drawn in blue are the same which we have seen in earlier stanzas.



A hand-drawn geometric diagram on lined paper. It features two triangles, TBC and LAM , which share a common vertex at point B (labeled as L). The top horizontal segment TB is drawn in blue ink. The right-hand segments BC and LM are drawn in red ink. The bottom-left segments LA and BM are also drawn in red ink. Vertical dashed lines connect point B to point A and point C to point M . The overall shape formed by points T, B, C, L, A, M resembles a bow-tie or a complex polygon.

As the position within the day change with khagola rotation about the pole point, we will take diurnal circle to generated points in reference to celestial horizon (for rising and setting reference). Project lines in khagola to meet ecliptic and celestial horizon as shown in figure. Figure 2 is zoomed in view of the triangle of our interest.

For approximation, Aksadrkkarma is considered as arcAB which is,

$$= R \sin(\text{local latitude}) \times \text{Moon's latitude} / R \cos(\text{local latitude})$$

$$= \text{ayanavalana} \times \text{Moon's latitude} / R$$

So,

Stanza 37:

चन्द्रस्जलं अर्कसग्निस्मद्भूषाया अपि या तमस्तहि ।

छादयति शशी सूर्यं शशिनं महती च भूछाया ॥ ३७ ॥

A Brief on ARYABHATIYA

"The Moon is water, the Sun is fire, the Earth is Earth, and what is called shadow is darkness. The Moon's eclipses the Sun and the greatest shadow of the Earth eclipses the Moon"

Here, Moon is water to meant to show some of the observational functions of Moon. At night Moon is seen and it cold at night like the touch of water. Also Moon reflects the light of Sun as water does. Aryabhata also mentioned that light doesn't pass through Moon and the shadow is cast on Earth during eclipse by which he also meant it as opaque solid.

Sun is a fire ball as we know. Earth is made of Earth. Moon's eclipses the Sun during solar eclipse and same way Earth on Moon during lunar eclipse.

Stanza 38:

स्फुटशशिमासान्ते अर्कं पातऽसन्नस्यदा प्रविशति इन्दुस् ।

भूछायां पक्षान्ते तदा अधिकऊनं ग्रहणमध्यम् ॥ ३८ ॥

"When at the end of lunar month, the Moon, lying near a node of the Moon, enters the Sun, or, at the end of lunar fortnight, enters the Earth's shadow, it is more or less the middle of an eclipse"

It is to be noted that Aryabhata stated clearly the role of Rahu and Ketu as the time of conjunction of Sun and the Moon as the middle of solar eclipse and the time of opposition of the Sun and the Moon as the middle of lunar eclipse. He also clearly states that as more or less which means that it is approximate. It indicates the fact that due to parallax, the actual time of conjunction is not exactly the same as that of geocentric conjunction.

Stanza 39:

भूरविविरं विभजेत्भूगुणितं तु रविभूविशेषेण ।

भूछायादीर्घत्वं लब्धं भूगोलविष्कम्भात् ॥ ३९ ॥

"Multiply the distance of the Sun from the Earth by the diameter of the Earth and divide by the difference between the diameters of the Sun and the Earth: the result is the length of the shadow of the Earth from the diameter of the Earth"

Formula shared is,

Length of Earth's shadow = (Sun's distance x Earth's diameter)/(Sun's diameter - Earth's diameter)

This method of calculating shadow length is called as pradipacchaya karma (Lamp and shadow method). We have also seen this in chapter 4 Ganita pada stanza 15.

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We will see the same again in correlation with this stanza.

Refer image. Let us consider a lamp post first which is installed at point B on ground and lamp at top is at point A. So, height of the light source is AB. Let us take gnomon vertically installed at point D and its tip as point C. Height of the gnomon is CD. Now, join BD and extend the line further till it intersects with the line made by point A & C. Let this intersecting point be E (tip of the shadow). So, we have three triangles, ABE, AFC & CDE. All these three mentioned triangles are similar triangles. So, as the formula given by Aryabhata in Ganita pada stanza 15,

Length of the shadow = Distance between the gnomon & light source x height of the gnomon / (height of the light source – height of the gnomon)

Refer our formula shared in this stanza.

Stanza 40:

छायाअग्रचन्द्रविवरं भूविष्कम्भेण तत्समभ्यस्तम् ।

भूछायया विभक्तं विद्यात्तमसस्वविष्कम्भम् ॥ ४० ॥

"Multiply the difference between the length of the Earth's shadow and the distance of the Moon by the Earth's diameter and divide by the length of Earth's shadow: the result is the diameter of the Tamas"

In last stanza, we saw the length of Earth's shadow. Here, we will see the diameter of Earth's shadow at Moon's distance which is called as Tamas.

Diameter of Tamas = (length of Earth's shadow - Moon's distance) x Earth's diameter / length of Earth's shadow

This is from similar triangle concept shared in Ganita pada. Refer image below. Let OA be the distance between Sun and Earth. OS be the semidiameter of Sun, AE be the semidiameter of Earth and BM is the semidiameter of Tamas at Moon's distance AB. It is to be noted that position of Moon while entering Tamas may be slightly out of BM line as it will follow elliptical path.

Now from figure,

We have two similar triangles, EAC and MBC.

So,

$$EA/AC = MB/BC$$

$$MB = EA \times BC/AC$$

$$\text{Diameter of Tamas} = 2MB$$

$$= 2 \times EA \times BC / AC$$

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$$= 2 \times EA \times (AC - AB)/AC$$

Diameter of Tamas = (length of Earth's shadow - Moon's distance) x Earth's diameter / length of Earth's shadow

This Tamas keeps moving with Sun's position around Earth. When Moon comes inside this Tamas we get to see lunar eclipse.

Stanza 41:

तद्शशिसम्पर्क अर्धकृतेस्शशिविक्षेपवर्गितं शोध्यम् ।

स्थिति अर्थ अस्य मूलं ज्ञेयं चन्द्रार्कदिनभोगात् ॥ ४१ ॥

"From the square of half the sum of the diameters of Tamas and the Moon, subtract the square of the Moon's latitude, and then take the square root of the difference: the result is known as half duration of the eclipse. The corresponding time is obtained with the help of the daily motions of Sun and Moon"

As we saw in earlier stanza on the size of Tamas at Moon's distance. When Moon enters Tamas, then eclipse starts and when it leaves, eclipse ends. So, parameters involved to find the duration of lunar eclipse are Moon's movement speed, diameter of Moon, Tamas size, Speed of Tamas(which depends on the position of Sun with respect to Earth) and Moon's latitude (why? Wait for next post). Let us see about this stanza now.

Half the duration of lunar eclipse in terms of arc minutes = $\sqrt{(\text{semi diameter of Tamas} + \text{semi diameter of Moon})^2 - (\text{Moon's latitude})^2}$

Half the duration of lunar eclipse in terms of ghatas = $60 \times \sqrt{(\text{semi diameter of Tamas} + \text{semi diameter of Moon})^2 - (\text{Moon's latitude})^2} / (\text{Moon's daily motion} - \text{Sun's daily motion})$

Upon iteration of the above formula, a saturated value for lunar eclipse duration can be calculated.

We will see in detail about this calculation in next post.

Before going into the calculation, we need to know the shadow concept and rough size of Moon inside Earth's shadow. Refer first image for Sun, Earth, Earth's shadow and Moon during a TOTAL lunar eclipse. We will see what is total lunar eclipse and partial lunar eclipse (there are few other types as well). Aryabhata has covered on total lunar eclipse in next stanza. We will see that also in this explanation. Next, after referring this image, it may seem like a simple calculation but let us see next on the parameters required to find the duration of lunar eclipse. Aryabhata's process is an iterative process.

A Brief on ARYABHATIYA

There are several different phases of a lunar eclipse. Refer second image. Now, you can understand, that Moon does not always pass through the center of Earth's shadow, so duration varies with the path of Moon with respect to the shadow circle.

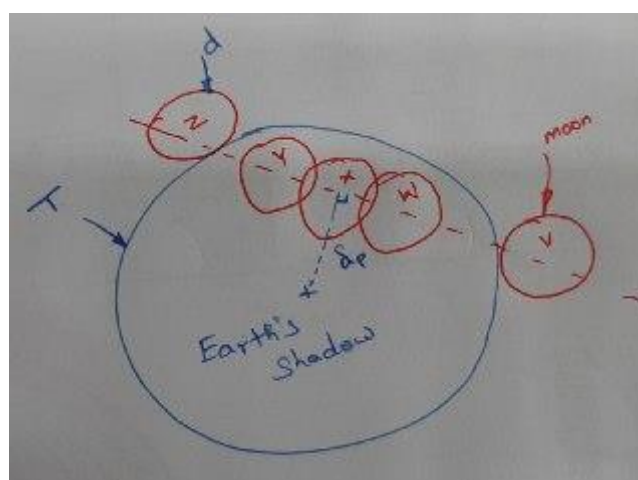
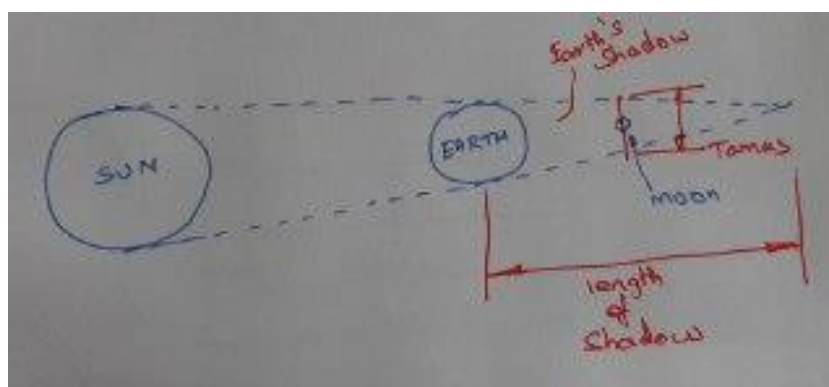
V: Entering Tamas (beginning of the eclipse)

W: Beginning of the total eclipse

X: Maximum of the eclipse (middle of the total eclipse)

Y: End of the total eclipse

Z: Exiting Tamas (end of the eclipse)



Time and duration are different. Time for the maximum of the eclipse(position X) can be calculated based on the alignment point of Sun, Earth and Moon (based on Rahu/ Ketu position with Moon and Sun). Lunar eclipse happens when Moon is closer to Rahu/ Ketu point and Earth shadow is also closer to Rahu/ Ketu(Sun is opposite to Rahu/ Ketu point). We will see about this in next post as a part of this calculation. For duration of eclipse, we need many parameters.

To find duration of partial eclipse and total eclipse, we need time of Y, Z, W and V. I will list down some values below which are required for calculation. Some of these values may vary with time but considering this for showing calculation.

A Brief on ARYABHATIYA

1. Radius of the Sun, R_s : 700 000 km
2. Radius of the Earth, R_e : 6 370 km
3. Radius of the Moon, R_m : 1 740 km
4. Earth - Sun distance (km), ES : 149 900 000 (varies, but considered this value for calculation purpose)
5. Earth - Moon distance (km), EM : 366 600 (varies, but considered this value for calculation purpose)
6. Moon orbits the Earth in a period of 29.53 days, relative to the Sun, or to the Earth's shadow.

Steps for our calculation are,

1. Earth's shadow length,
2. Tamas diameter,
3. Speed of Moon,
4. Extent of lunar eclipse and then
5. Duration

1. Earth's shadow length,

$SL = ES \times R_e \times 2 / (2 \times (R_s - R_e))$ (refer stanza on shadow length for formula)

$$= 149900000 \times 6370 \times 2 / (2 \times (700000 - 6370))$$

$$= 1376617.2166 \text{ km}$$

2. Tamas diameter,

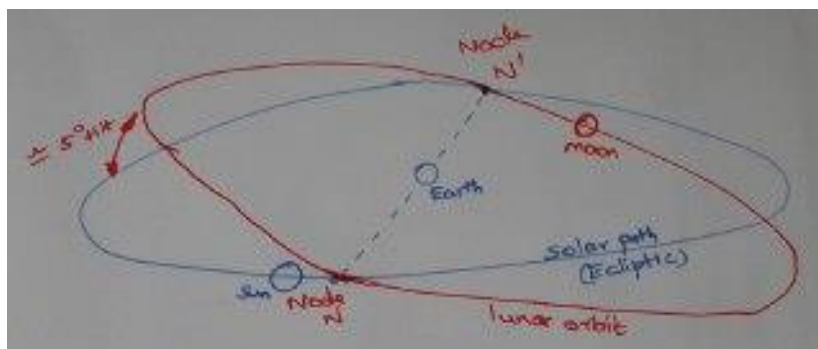
$$Dt = (SL - EM) \times R_e \times 2 / (SL)$$

$$= (1376617.2166 - 366600) \times 6370 \times 2 / 1376617.2166$$

$$= 9347.27 \text{ km}$$

3. Speed of Moon:

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Refer first image for Sun's and Moon's path. Interesting points are nodes N and N' (Rahu and Ketu). As Earth also rotates, these nodes also move around Earth. Earth's shadow moves in Sun's path but opposite side of Sun. Moon will come inside Earth's shadow when Earth's shadow and Moon meets at node. Note that there is an tilt with Moon's path and Sun's path. If this is not there, then we would have got lunar eclipse every lunar month. Now, since we are in the process of finding duration which is time duration, we need time factor in the equation. Since, lunar eclipse depends on the movement of Earth's shadow and Moon, we need to consider the relative motion of Moon with respect to Sun. The Moon orbits the Earth in a period of 29.53 days, relative to the Sun, or to the Earth's shadow. This period is the same as the interval between two successive Full Moons, or two New Moons. Now, we need the speed of Moon around Earth for an observer in Earth. If it takes 29.53 days for Moon to come around relative to Sun, which is 360deg (2xpi) in angle for a revolution at circumference of Moon's distance from Earth. Then,

Speed of Moon = Distance traveled/ Time taken

$$= 2 \times \pi \times \text{radius} / \text{Time for one revolution}$$

$$= 2 \times 3.14 \times \text{EM} / 29.53 \quad \text{in km/ days}$$

$$= 2 \times 3.15 \times 366600 / (29.53 \times 24) \quad \text{in km/hr}$$

$$= 3250.1144 \text{ km/hr}$$

$$= 54.1685 \text{ km/min}$$

Now for the duration of the eclipse, we need to find time from distance traveled and speed. Also note that Moon's speed is not constant but considered so for approximation. Moon orbits the Earth on an ellipse; its distance to the Earth can vary from 356 000 to 407 000 km and also speed varies as we have studied in Kepler's second law of planetary motion.

Duration of eclipse = distance travelled by Moon inside Tamas/ speed of Moon.

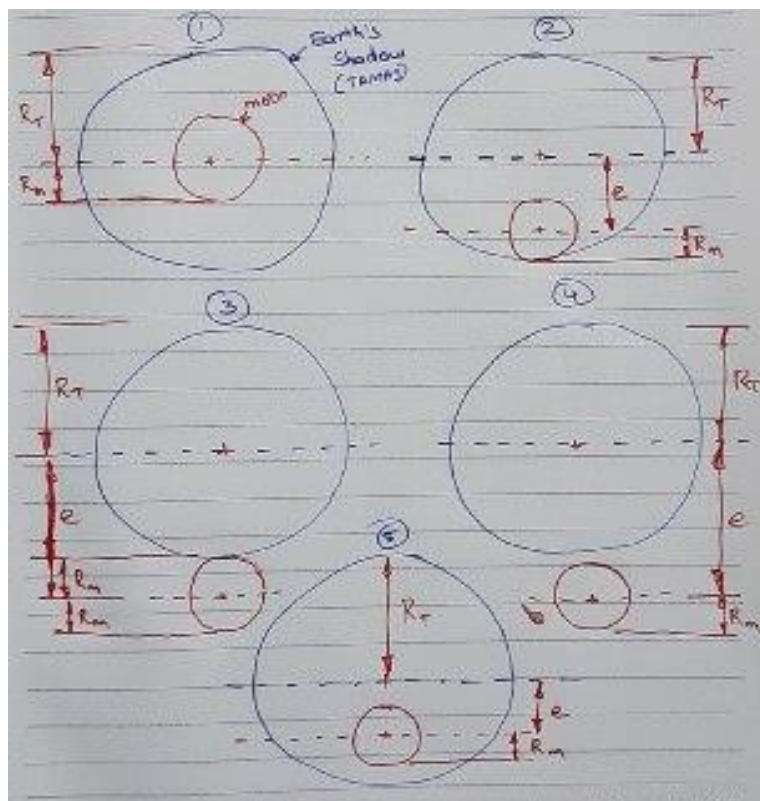
Aryabhata in his work has shared formula for the distance travelled and for relative speed he has given total revolution of planets.

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Here, we finding an approximate values as the accurate value also depends on parallax, speed of light and movement of Tamas, etc.,

To find distance travelled by Moon, we need to also know the extent of an eclipse.

4. Extent of lunar eclipse:



Refer second image where different scenarios are shown for understanding different conditions for an eclipse. This is the major parameter which changes among different eclipses which changes its duration. Aryabhata has shared extent of eclipse in upcoming stanzas. In earlier stanza he has shared on how to find distance travelled by Moon inside Tamas which we will see as a part of this post. For extent of lunar eclipse,

Case1- Moon passes thru the center of Tamas: this condition will result in maximum duration of the eclipse as Moon has to cross the maximum distance with Tamas which is its diameter. In this case eccentricity, e is 0.

Extent of lunar eclipse, $g = (R_t + R_m - e)/(2R_m)$

This is just an equation to find the extent of eclipse which is defined by the maximum and minimum conditions.

So for case1, $g = (R_t + R_m)/(2R_m) > 1$

Case2- Moon passes thru a path with one instance completely inside Tamas: Here, we get to see Total eclipse for a fraction of time and rest is partial eclipse.

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$$e = R_t - R_m$$

$$g = (R_t + R_m - R_t + R_m)/(2R_m)$$

$$g = 1$$

So, from case 1 and case 2 it is clear that if g is greater than 1 then we get to see total eclipse.

Case3- Moon passes outside Tamas but touches at one point: In this condition, Moon just touches and go without even a partial eclipse. So,

$$e = R_t + R_m$$

$$g = (R_t + R_m - R_t - R_m)/(2R_m)$$

$$= 0$$

So, if g is between 0 to 1, it is partial eclipse and if g is greater than 1 then Total eclipse.

Case4- Moon passes completely outside Tamas: Here, e is greater than $R_t + R_m$ to g will be negative. If extent of an eclipse, g is negative then no eclipse.

Case5- Moon at the position in between somewhere in Tamas:

Now, let us see a case where Moon passes inside Tamas. Now, refer first image. Eccentricity, e depends on the path of Moon with Tamas in path of Sun. Tamas moves as Earth moves around Sun. Considering speed of Tamas is comparatively less to speed of Moon, we will see how to find eccentricity, Moon's latitude and distance travelled by Moon inside Tamas.

In image, I have marked R_t as R and R_m as r . Here, there are two right angle triangle with common side as Moon's latitude. Half travel distance of Moon for eclipse is AB (including total and partial) and AB' is half travel distance of Moon for total eclipse. We will write formula for these distances in terms of R_t , R_m and e (eccentricity or Moon's latitude). Please note that first contact point on Tamas gives Moon's latitude which is related to eccentricity. For case1, first contact is at 0deg lat of Moon. For case4, at 90deg Moon's lat. For case2, it can be calculated based on radius of Tamas and Moon.

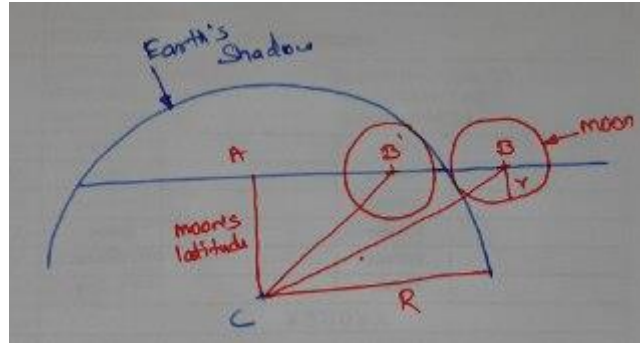
Aryabhata shared in terms of Moon's latitude, e , as it can be observed. We will see this in next post. It is to be noted that our ancestors are keen observers. For find Moon's first contact latitude, you need to observe Moon's path on that day and then with reference to path the first contact point needs to be observed.

$$AB' = \sqrt{CB'^2 - e^2}$$

$$CB' = R_t - R_m$$

$$\text{So, } AB' = \sqrt{(R_t - R_m)^2 - e^2}$$

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Refer formula in stanza 41 now which is same as above formula. For duration, we need to divide this by speed of Moon relative to Sun. For a sample calculation on extent and duration, we will take lunar eclipse happening next year on 26th May 2021.

Lunar eclipse on 26th May 2021 has magnitude of 1.009 at Maximum eclipse which is at 16.49. I had mentioned about iteration process earlier. Finding magnitude is a iterative process. We will see that after seeing next two stanzas. Magnitude depends on the true longitude of Moon and node. Nodes are invisible and position of those can be determined only by calculation. Hence they are called Demons (Rahu and Ketu) as they are invisible. There are different methods to find like from ahargana and revolution made by nodes and Moon or from earlier eclipse details or observation of Sun and Moon position or first, second, third and fourth contact points during eclipse. Also, we need the time of Maximum eclipse which can also be determined from one of the above methods.

Total eclipse duration:

Extent of lunar eclipse or Magnitude, $g = (R_t + R_m - e) / (2R_m)$

$$1.009 = (9347.27/2 + 1740 - e) / (2 \times 1740)$$

$$e = 2902.18 \text{ km}$$

$$AB' = \sqrt{(R_t - R_m)^2 - e^2}$$

$$= \sqrt{(4673.63 - 1740)^2 - 2902.18^2}$$

$$= 427.52 \text{ km}$$

Half duration of Total eclipse = $427.52 / \text{speed of Moon}$

$$= 427.52 / 54.1685$$

$$= 8 \text{ min}$$

Total duration of Total eclipse = 16min

Partial eclipse duration:

$$e = 2902.18 \text{ km}$$

A Brief on ARYABHATIYA

$$AB = \sqrt{(R_t + R_m)^2 - e^2}$$

$$= \sqrt{(4673.63 + 1740)^2 - 2902.18^2}$$

$$= 5719.44 \text{ km}$$

$$\text{Half duration of partial eclipse} = 5719.44 / \text{speed of Moon}$$

$$= 5719.44 / 54.1685$$

$$= 105.58 \text{ min}$$

$$= 1 \text{ hr } 46 \text{ min}$$

$$\text{Total duration of partial eclipse} = 2 \times 106.58$$

$$= 213.16 \text{ min}$$

$$= 3 \text{ hr } 33 \text{ min}$$

So, based on above calculation following are the timelines,

Maximum eclipse time = 26-05-2021 16:49 at Mumbai, India

Start of partial eclipse (-105.58min) = 26-05-2021 15:03

Start of total eclipse (-8min) = 26-05-2021 16:41

End of total eclipse (+8min) = 26-05-2021 16:57

End of partial eclipse (+105.58min) = 26-05-2021 18:35.

Refer images for more accurate results. Variation in time is due to other factors like parallax, speed of light, exact values of EM, Rm, etc., not considered. Aryabhata has shared formula for parallax also.

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Event	UTC Time	Time in Mumbai*	Visible in Mumbai
Penumbra Eclipse begins	26 May, 08:47:39	26 May, 14:17:39	No, below the horizon
Partial Eclipse begins	26 May, 09:44:58	26 May, 15:14:58	No, below the horizon
Full Eclipse begins	26 May, 11:11:26	26 May, 16:41:26	No, below the horizon
Maximum Eclipse	26 May, 11:18:42	26 May, 16:48:42	No, below the horizon
Full Eclipse ends	26 May, 11:25:54	26 May, 16:55:54	No, below the horizon
Partial Eclipse ends	26 May, 12:52:23	26 May, 18:22:23	No, below the horizon
Penumbra Eclipse ends	26 May, 13:49:44	26 May, 19:19:44	Yes

Stanza 42:

चन्द्रव्यास अर्धऊनस्य वर्गितं यत्तमस्मय अर्धस्य ।

विक्षेपकृतिविहीनं तस्मात्मूलं विमर्द अर्धम् ॥ ४२ ॥

"Subtract the semi diameter of Moon from the semidiameter of that Tamas and find square of that difference. Diminish that by the square of Moon's latitude and then take the square root of that and thus obtained is the duration of totality of eclipse"

That is,

Half duration of total eclipse in arc minutes= $\sqrt{(T+R_m)^2 - (\text{Moon's lat})^2}$

Half Time duration of total eclipse = $(\sqrt{(T+R_m)^2 - (\text{Moon's lat})^2})/(\text{Moon's daily motion} - \text{Sun's motion})$

We have seen this calculation in earlier post.

This gives approximate result but upon iteration for Moon's latitude, accurate results can be obtained.

Stanza 43:

तमसस्विष्कम्भ अर्ध शशिविष्कम्भ अर्धवर्जितं अपोह्य ।

विक्षेपात्यक्षोषं न गृह्यते तत्साशाङ्कस्य ॥ ४३ ॥

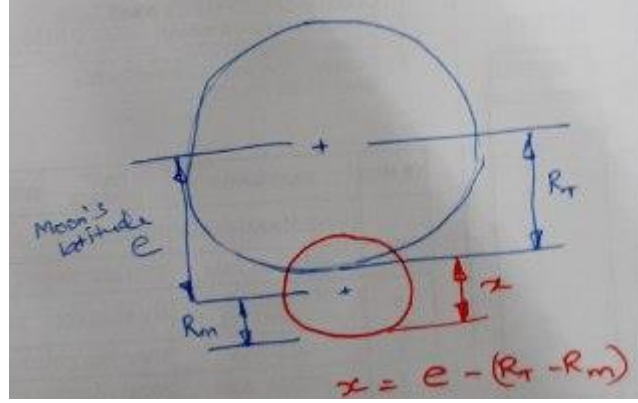
"Subtract the Moon's semidiameter from the semidiameter of Tamas; the subtract whatever is obtained from the Moon's latitude: the result is the part of the Moon not eclipsed by the Tamas"

That is,

A Brief on ARYABHATIYA

Length of Moon's diameter not eclipsed = Moon's latitude - (semidiameter of Tamas - semidiameter of Moon)

Refer image.



Stanza 44:

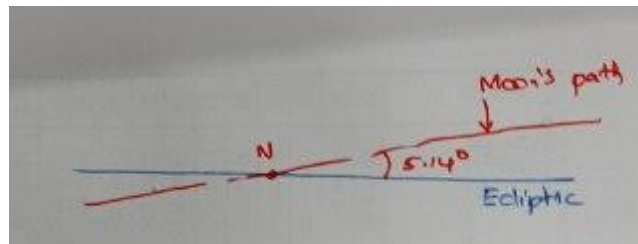
विक्षेपवर्गसहितास्थितिमध्यातिष्ठवर्जितामूलम् ।

सम्पर्क अर्धाशोधं शेषस्तात्कालिकस्रासस् ॥ ४४ ॥

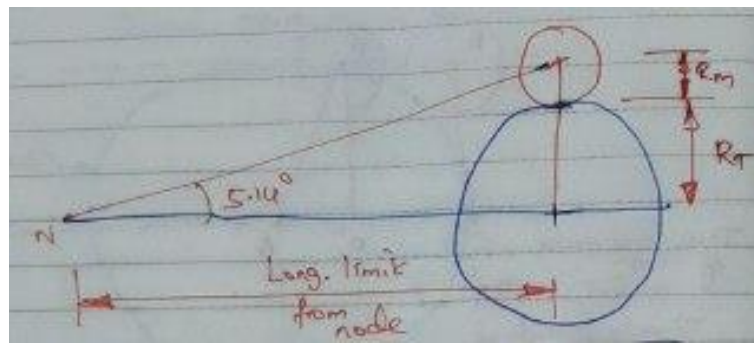
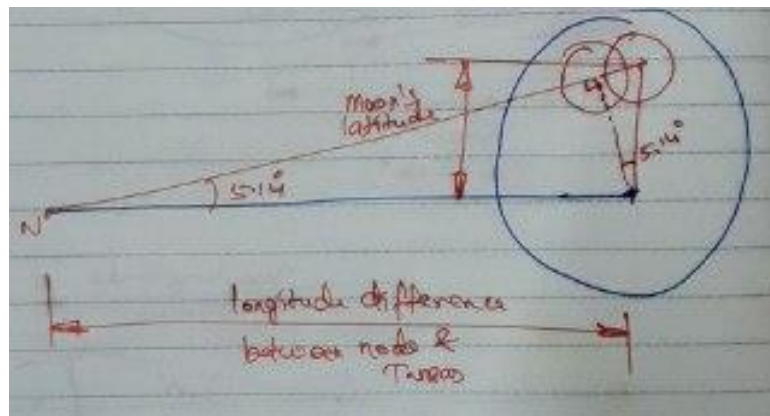
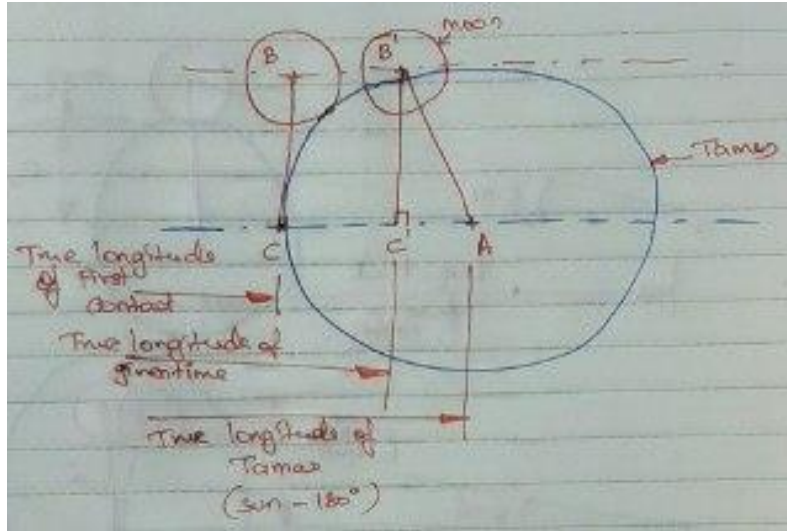
"Subtract the ista from the semi duration of the eclipse; to the square of that difference add the square of Moon's latitude at that time; and take square root of this sum. Subtract that square root from the sum of the semidiameter of the Tamas and Moon: the remainder is the square of the eclipse at the given time"

It should have Aryabhata theorem instead of Pythagoras theorem. Even in Aryabhatiya alone, Aryabhata had used this multiple times. Let us now see the amount of eclipse at a given time as shared by Aryabhata. Let us also see the iteration to find maximum eclipse, Moon's latitude at maximum eclipse which I mentioned that I will explain later.

Now refer first image shared. Moon's orbital path is tilted at an angle of about 5.14deg from the ecliptic (path of Sun and Tamas). Now, the intersection of these orbits are nodes. Since, the angle is very low, for finding duration of eclipse, many may have considered the path of Moon and ecliptic as parallel to arrive at simple formula. But, since Moon's latitude will be small from Tamas, this angle defines the Moon's latitude if we know true longitude of Moon's and Sun or Tamas.



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Refer second image. Here, we will first see what is shared by Aryabhata in this stanza. Eclipse is measured based on the intersection of Moon and Tamas (thickness of the eye shape). Refer Ganita pada, stanza 18, formula for amount of intersection is shared by Aryabhata. Based on the formula,

Measure of eclipse = $R_t + R_m - AB'$

$$= (R_t + R_m) - \sqrt{B'C'^2 + (AC - CC')^2}$$

Here,

$B'C'$ = Moon's true latitude at a given time

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AC = difference in true longitude of Tamas and Moon's first contact point

CC' = difference in true longitude of Moon from first contact and given time

Now, coming back to Moon's latitude,

Refer third image, it can be seen that Moon's latitude depends on the true longitude of Node, tilt in orbit and true longitude of Moon.

$\tan(5.14) = \text{Moon's latitude} / (\text{True longitude of Moon} - \text{True longitude of Moon})$

Also, note that the possibility of lunar eclipse is also limited to a particular true longitude of Moon as shown in fourth image. Since, for lunar eclipse, we need to find true longitude of Moon, Tamas and Node, Aryabhata has shared the method of calculating True longitude in Kalakriya pada.

Stanza 45:

मध्याह्नौक्रमगुणितसक्षस्दक्षिणतस् अर्धविस्तरहतस्दिक् ।

स्थिति अर्धात् अर्केन्द्रोस् त्रिराशिसहितायनात्स्पर्शे ॥ ४५ ॥

"Multiply the Rsine of the hour angle by the latitude and divide by the radius: the result is aksavalana. Its direction is south. Making use of the semiduration of the eclipse, calculate the longitude of Sun and Moon for the time of first contact. Increase that longitude by three signs and by the Rsine of the Sun's greatest declination and dividing calculate the Rsine of the corresponding declination: this ayanavalana, time of first contact"

Aksavalana means deflection due to latitude. First half of the formula shared by Aryabhata here is on aksavalana and the second half is on Ayanavalana which is deflection due to the ecliptic which is used during eclipses. We saw the deflection due to the ecliptic in the earlier stanza. In addition to the formula we saw earlier, Aryabhata also shared the transformation to celestial equator.

$\text{Aksavalana} = \text{Rsin}(\text{Hour angle}) \times \text{Rsin}(\text{local latitude}) / R$

$\text{Ayanavalana} = \text{Rsin}(\text{long. of eclipsed body}) \times \text{Rsin}(\text{greatest declination}) / R$

Greatest declination considered by Aryabhata is 24deg.

Above formula gives correlation between local angle and that of the eclipse to calculate the first contact time. After that by iteration, that is, by increasing Moon's longitude step by step we can find the maximum value of measure of eclipse and then the time of maximum eclipse can be found. Refer earlier stanza where I have shared the condition for first contact.

First the day of eclipse is determined by the revolution numbers then longitude of eclipsed object is found at the time for intersection. Precise value is identified by changing the longitude closer to the eclipse zone and then comparing with the

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condition shared to set exact time of start, max and end of eclipse. If we consider UTC time zone and transform to that longitude then we will get time against UTC.

Stanza 46:

प्रग्रहणान्ते धूम्रखण्डग्रहणे शशी भवति कृष्णस् ।

सर्वग्रासे कपिलस्सकृष्णताम्रस्तमस्मध्ये ॥ ४६ ॥

"At the beginning of its eclipse, Moon is smoky; when half obscured, it is black; when totally obscured, it is tawny; when far inside the shadow, it is copper coloured with blackish tinge"

Stanza 47:

सूर्येन्दुपरिधियोगे अर्क अष्टमभागस् भवति अनादेश्यस् ।

भानोष्मास्वरभावात्सुअच्छतनुत्वात् शशिपरिधेस् ॥ ४७ ॥

"When the disc of Sun and Moon comes into contact, a solar eclipse should not be predicted when it amounts to one-eighth of Sun's diameter on account of brilliancy of Sun and transparency of Moon"

We all know that looking at bright sun is dangerous to eyes and may result in permanent blindness. Aryabhata here in Aryabhatiya explained about lunar eclipse and mentioned precautions for Solar eclipse in this stanza.

Stanza 48:

क्षितिर्वियोगादिनकृत्रविन्दुयोगात् प्रसाधयेत् इन्दुम् ।

शशिताराग्रहयोगात्तथा एव ताराग्रहास्सर्वे ॥ ४८ ॥

"Sun has been determined from the conjunction of Earth and Sun, Moon from the conjunction of Sun and Moon, and all other planets from the conjunction of planets and Moon"

It can be noted that for all other planet it is from Moon's reference at planets can be observed at night with Moon. Also, ancient Indians knew how to pin point planets at night without any aid like telescopes.

Here, in this stanza, the position of the planets can be identified based on the revolution number and the angular distance from their respective conjunction. Refer earlier stanza on revolution number in Githika pada where we have already seen what is shared in this stanza.

Stanza 49:

सतसत्त्वानसमुद्रात्समुद्धृतं ब्रह्मणस्प्राप्तादेन ।

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सद्भानौत्तमरत्नं मया निमग्नं स्वमतिनावा ॥ ४९ ॥

"By the grace of Brahma, the precious jewel of excellent knowledge has been brought out by me by means of the boat of my intellect from the sea of true and false knowledge by diving deep into it"

Stanza 50:

आर्यभटीयं नाम्ना पूर्वं स्वायम्भुअवं सदा नित्यम् ।

सुकृतऽयुषोऽग्राणां कुरुते प्रतिकञ्चुकं यस्यस्य ॥ ५० ॥

"This work, Aryabhatiya by name, is the same as ancient svayambhuva which was revealed by Svayambhu and such it is true for all times. One who imitates it or finds fault with it shall lose his good deeds and longevity"

Concept shared in Aryabhatiya was actually from Svayambhuva which would have been followed by Ancient Indian astronomers like Aryabhata. Here in Aryabhatiya, Aryabhata shared numbers based on his observation for us to simplify and find what we need and help us to do such observation.

With this Aryabhatiya is over. Thanks to Aryabhata and his commentators for sharing this in a way which was easy even for me to understand.